The passivity-based stabilization of switched nonlinear systems under asynchronous switching

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Abstract: In this paper, we address the stabilization issue of switched nonlinear systems with passive and non-passive subsystems in which the controllers are switched asynchronously with the switching of system modes. For any given average dwell time, any given passivity rate, and any admissible switching delay, we design feedback mode-dependent controllers of subsystems to achieve exponential stabilization. An example is provided to verify the efficiency of the proposed method.

Keywords: Switched nonlinear systems; Passivity; Average dwell time; asynchronous switching

1 Introduction

Switched systems have drawn considerable attention in the last decades since a large class of practical systems can be modeled as switched systems [1, 2]. The main concern in the study of switched systems is the issue of stability, which is very difficult to deal with due to the hybrid nature of switched systems operation [3, 4]. The existence of a common Lyapunov function of all the subsystems was shown to be a necessary and sufficient condition for stability of a switched system under an arbitrary switching law [1]. In practice, most switched systems do not possess a common Lyapunov function, yet they still may be asymptotically stable under some properly chosen switching law. Multiple Lyapunov function method is a generally used tool when studying the asymptotic stability problem of switched systems under a certain switching law [5]. However, when applying this method, a non-increasing condition of the switched-on sequence is required and this is usually hard to check in general. In [6], a necessary and sufficient condition is given in terms of multiple generalized Lyapunov-like functions, which removes this assumption. There are many available results of construction of stabilizing switching sequences [7]. The average dwell time method is also an effective tool, under which some switching laws can be chosen to achieve asymptotic stability [8]. Furthermore, in [9], the stability property of switched linear systems consisting of both Hurwitz stable and unstable subsystems was studied for the first time under the average dwell time scheme. This result was extended to the stability analysis of systems with passive and non-passive subsystems [10].

On the other hand, passivity, developed by Willems [11] and further extended by Hill and Moylan [12], is a very useful system property. Passivity of nonlinear systems has attracted great interest in the control area. Also, there are many works concerning passivity based control [13, 14]. Passivity is important not only for smooth systems, but also useful for switched systems [15, 16]. A passivity-based design method for switched systems was proposed [17]. Passivity theory and the corresponding passivity-based stability analysis using multiple storage functions and multiple supply rates were set up in [18]. Passivity-based control was discussed [19, 20]. The uniform stability conditions were derived for decomposable dissipative discrete-time switched systems [21]. A dissipativity-based switching adaptive control strategy for uncertain systems was considered in [22].

However, it is worthy pointing out that the majority of the results mentioned above were based on an ideal assumption that the controller has instant access to the switching signal of the system. In such a case, the switching between the controller and the system is synchronous, which is quite impractical. The necessities of considering asynchronous switching for efficient control design have been shown in [23]. Asynchronous switched control of switched linear systems with average dwell time was the concern of the paper [24]. However, in [25, 26], a common function was chosen for the running time of the subsystem with both the matched controller and the unmatched controller. Therefore, at the switching times of the controller, the function is required to be continuous. Ref. [27] addressed stability of time-delay feedback switched linear systems in which time delays appear in both the feedback state and the switching signal of the switched controller. Up to now, to the best of our knowledge, for switched systems, the asynchronous stabilization control problem using passivity has not been studied so far.

Motivated by the above discussion, in this paper, we investigate the stabilization problem of switched nonlinear systems under asynchronous switching. A sufficient condition is given to achieve exponential stabilization for switched nonlinear systems consisting of passive...
subsystems and non-passive subsystems by using an average dwell time approach. Compared with the existing results on stabilization problem for switched nonlinear systems, the results of this paper have two distinct features. Firstly, we study the problem for a more general class of switched nonlinear systems, while the existing literatures [23], [25] mainly focus on switched linear systems. Secondly, for any given average dwell time, any given passivity rate, and all admissible switching delay, controllers can be designed to solve the problem, while the classical average dwell time is computed by Lyapunov functions of the subsystems and obviously can not be arbitrarily given.

2 Problem statement and preliminaries

In this paper, we consider the switched nonlinear system of the form [10]

\[
\begin{align*}
\dot{x} &= f_{\sigma(t)}(x) + g_{\sigma(t)}(x)u_{\sigma(t)}, \\
y &= h_{\sigma(t)}(x),
\end{align*}
\]

(1)

where \( x(t) \in \mathbb{R}^n \) is the state, \( u_i(x) \in \mathbb{R}^r \) and \( h_i(x) \in \mathbb{R}^r, i = 1, \ldots, m \) are the control input and the measurement output of the \( i \) th subsystem, respectively. \( f_i(x), g_i(x) \) are smooth vector fields. Further, it is assumed that \( f_i(0) = 0 \) and \( h_i(0) = 0 \). \( \sigma(t) : [0, \infty) \rightarrow \{1, 2, \ldots, m\} \) is the switching signal. The switching sequence \( \{x_0; (t_0, t_0), (i_1, t_1), \ldots, (i_k, t_k), \ldots\} \mid i_k \in I_k, k \in \mathbb{N} \}

means that the \( i_k \) th subsystem is active when \( t \in [t_k, t_{k+1}) \).

We assume that the state of the system does not jump at the switching instants and that only finitely many switches can occur in any finite interval.

The control input \( u_{\sigma(t)} \) in (1) is used to achieve system stability or certain performances for certain switching signals, and usually in the literature, a common assumption is that the switches of the control input coincide with those of the system, which is hard to satisfy in practice. If the time lag of switched controllers to systems (asynchronous switching) is \( \tau(t) \), the control input will become \( u_{\sigma(t-\tau(t))} \), and hence the resulting closed-loop system is given by

\[
\begin{align*}
\dot{x} &= f_{\sigma(t)}(x) + g_{\sigma(t)}(x)u_{\sigma(t-\tau(t))}, \\
y &= h_{\sigma(t)}(x),
\end{align*}
\]

(2)

where \( \tau(t) : \mathbb{R} \rightarrow [0, \tau_\Omega] \) is the switching delay satisfying the switching delay condition

\[
0 \leq \tau_i(t) \leq \tau_\Omega < t_{i+1} - t_i, \quad i \in \mathbb{N},
\]

(3)

where \( \tau_\Omega \) is a nonnegative real number. Obviously, the unmatched controllers in the loop, together with the switching signals in the case of synchronous switching, may cause instability or a worse performance for the underlying system.

The objective is to design controllers that can exponentially stabilize the switched system (1) with passive and non-passive subsystems under switching signals.

We employ the merging signal technique in [27] to deal with the mismatched switching signal. Similarly, we create a virtual switching signal \( \sigma'(t) : [0, \infty) \rightarrow I \times I \) as follows \( \sigma'(t) = (\sigma_1, \sigma_2) \). The merging action is denoted by \( \oplus \) such that \( \sigma'(t) = \sigma_1 \oplus \sigma_2 \). The definition implies that the set of switching times of \( \sigma'(t) \) is the union of the sets of switching times of \( \sigma_1 \) and of \( \sigma_2 \).

Subsequently, we introduce the definition of the exponential stability.

Definition 1 [9] For a switching signal \( \sigma(t) \), the switched system (1) is said to be globally exponentially stable with stability degree \( \lambda \geq 0 \) if

\[
\|x(t)\| \leq \alpha \exp(-\lambda(t-t_0))\|x(t_0)\|
\]

holds for all \( t \geq t_0 \) and a known constant \( \alpha \).

The notion of passivity is given as follows.

Definition 2 [12] The \( i \) th subsystem for the system (1) is said to be passive if there exists a \( C^1 \) function \( V_i(x) \) satisfying

\[
a_i(\|x\|) \leq V_i(x) \leq a_2(\|x\|), \quad x \in \mathbb{R}^n \text{ where } a_1(\cdot) \text{ and } a_2(\cdot) \text{ are class } K \text{ functions, such that}
\]

\[
L_{h_i}V_i(x) + L_{g_i}V_i(x)u_i \leq h_i(x)u_i
\]

for all \( x \in \mathbb{R}^n, u_i(x) \in \mathbb{R}^r \) and \( h_i(x) \in \mathbb{R}^r \).

A necessary and sufficient condition for the system to be passive is that the system satisfies the KYP property [11], i.e.

\[
L_{h_i}V_i(x) \leq 0, \quad L_{g_i}V_i(x) = h_i(x).
\]

Here we do not need passivity of all subsystems. For simplicity, we classify the subsystems into two groups. The \( i \) th subsystem is passive for \( i \in I_p \subset I \) and non-passive for \( i \in I_n = I - I_p \). In order to study the stabilization problem using passivity, at least one subsystem is required to be passive, i.e. \( I_p \neq \Phi \). Also, the activation time ratio of passive subsystems and non-passive subsystems plays a crucial role.

Definition 3 [10] For any \( 0 \leq T_1 < T_2 \), let \( T_{p[I_1, I_2]} \) denote the total time when the passive subsystems are active on \( [T_1, T_2] \), then \( r_{p[I_1, I_2]} = \frac{T_{p[I_1, I_2]}}{T_2 - T_1} \) is called the passivity rate of the switched system (1).

Now, we pay special attention to the subsystems (2) with the form

\[
\begin{align*}
\dot{x} &= f_i(x) + g_i(x)u_j, \\
y &= h_i(x), \quad i, j \in I,
\end{align*}
\]

(4)

where the controllers are switched asynchronously with the switching of system modes, i.e. the \( i \) th subsystems uses the controller of the \( j \) th subsystem in \( t \in [t_k, t_k + \tau_j(t_k)] \),

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$i_k = i$. Now, recall [1] that a switching signal $\sigma(t)$ is an average dwell-time signal if the number of switches in any interval $[t, t+\tau]$, denoted by $N_\sigma(t, t)$, satisfies $N_\sigma(t, t) \leq N_0 + \frac{t-t_0}{\tau_a}$ for some constant $N_0 \geq 1$; $N_0$ is called a chatter bound. Denote by $S_{ave}[\tau_a, N_0]$ the class of switching signals with average dwell-time $\tau_a$ and chatter bound $N_0$.

By Lemmas 1 and 2 in [27], the Lemma 1 can be easily obtained.

**Lemma 1.** Given $\sigma_1 \in S_{ave}[\tau_a, N_0]$ and $\sigma_2 = \sigma_1(t-t_*(t))$, then, it has $\sigma_2 \in S_{ave}[\tau_a, N_0 + \frac{\tau_m}{\tau_a}]$, $\sigma' \in S_{ave}[\tau_a, N_0]$, where $\tau_a = \frac{\tau_a}{2}$, $N_0 = 2N_0 + \tau_m$.

Next, we introduce the concept of exponentially small-time norm-observability for nonlinear systems, **Definition 4** [10] The nonlinear system

$$\dot{x} = f(x)$$

$$y = h(x)$$

is said to be exponentially small-time norm-observable with degree $\lambda$ if there exists $\delta > 0$ such that when $y(t+s) \leq \delta$ holds for some $t \geq t_0$, $\tau > 0$ and $0 < s \leq \tau$, we have $\|y(t+s)\| \leq c e^{-\lambda \tau} \|y(t)\|$.

3 Main results

In this section, we design mode-dependent controllers for subsystems to stabilize the system (1) by using the average dwell time approach.

**Theorem 1** Consider the switched system (1) and let $\tau_a$, $r$, and $\tau_d$ be any given average dwell time, passivity rate and admissible switching delay satisfying $\tau_a < r \tau_d$, respectively. Suppose that there exist $C^1$ positive definite functions $V_i(x)$, $V_{i,j}(x)$ and constants $a_i > 0$, $a_2 > 0$, $a_3 > 0$, $\mu \geq 1$, and $\delta > 0$, such that,

(i) for $i \in I_n$, there exists a constant $\lambda > 0$ satisfying

$$L_j \dot{V}_i(x) \leq \lambda \dot{V}_i(x).$$

(ii) for $i \in I_p$, it holds that

$$L_j \dot{V}_i(x) \leq 0, \quad L_{i,j}(x) = h^T_i(x).$$

In addition, the passive subsystem with $u_i = 0$ is assumed to be exponentially small-time norm-observable with constants $\lambda^*$ and $c$ satisfying $\lambda \geq \frac{1}{2} \lambda^* > 0, c \leq \sqrt{a_2}$, where

$$\lambda^* = \frac{\tau_a}{r \tau_d - \tau_0} (\lambda_2 + (1-r)(\lambda_1 + \lambda_a) + 2 \ln \mu)$$

for some constant $\lambda_2 > 0$.

Further, for $\forall x \in \mathbb{R}^n$, $\forall i, j \in I$, the conditions

$$a_i \|x\| \leq V_{i,j}(x) \leq a_2 \|x\|, \quad \|V_{i,j}(x)\| \leq a_3 \|x\|,$$

$$V_{i,j}(x) \leq \mu V_{i,j}(x),$$

hold. And there exists a real number $\lambda > 0$, such that

$$\dot{V}_{i,j}(x(t)) = \frac{dV_{i,j}(x)}{dt} (f_i(x) + g_i(x)u_j) \leq \lambda \dot{V}_{i,j}(x),$$

hold for the controllers $u_j(x) = -k_j (L_{i,j}(x))^T$, $i \in I_n$, $j \in I_p$.

Design controllers

$$u_i(x) = \begin{cases} -k_i (L_{i,i}(x))^T, & i \in I_p, \\ 0, & i \in I_n, \end{cases}$$

where $k_i = \lambda^* \frac{1}{L_{i,i}(x)} \|L_{i,i}(x)\| > 0$. Then, the switched system (1) is exponentially stable under any switching signals with the average dwell time $\tau_a$, passivity rate $r \tau_d \geq r$ and admissible switching delay $\tau_d$.

**Proof** Define the set

$$S = \left\{ t : \left\|L_{i,i}(x(t))\right\| \leq \delta, \quad i \in I_p \right\}.$$ We now split the proof into two cases of $S = \emptyset$ and $S \neq \emptyset$.

Case 1: $S = \emptyset$.

Let $\sigma^*(t) = \sigma(t) \oplus \sigma(t-\tau(t))$, and the virtual switching times $t_{s_j} = t_0 < t_{s_1} < \ldots < t_{s_j} < \ldots$. When the $i$th subsystem is active, it follows that the time derivative of the $V_{i,j}(x)$ along the trajectory of the subsystem $\Sigma_{i,j}$ is

$$\dot{V}_{i,j} = \frac{\partial V_{i,j}(x)}{\partial x} f_i + \frac{\partial V_{i,j}(x)}{\partial x} g_i u_j, \quad \forall i, j \in I.$$ (13)

By definition to the passivity rate, the firstly active subsystem is passive, i.e. $i_0 \in I_p$. Substituting the controllers (11) into (12), we obtain

$$\dot{V}_{i_0} = \frac{\partial V_{i_0}(x)}{\partial x} f_0 + \frac{\partial V_{i_0}(x)}{\partial x} u_0 \leq -\lambda V_{i_0}(x(t_0)).$$

Similarly, when $i \in [t_{s_j}, t_{s_{j+1}})$, $i_j \in I_n$, $t_{s_{j+1}} = t_{s_j} + \tau(t_{s_j})$, $k \geq 1$, the non-passive subsystems use the
controller of the passive subsystems, we obtain
\[ \dot{V}_{i_k, i_{k+1}} \leq \lambda_i V_{i_k, i_{k+1}+1}, \quad i_k \in I_n \] and
\[ V_{i_k, i_{k+1}} \leq \exp(\lambda_i (t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}), \quad (14) \]
when \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_n \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we obtain \( \dot{V}_{i_k, i_{k+1}} \leq 0 \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (15) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 1 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq 0 \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
Choose the piecewise function
\[ V(x(t)) = V_{i_k, i_{k+1}}(x(t)), \quad t \in [t_{j_k}, t_{j_{k+1}}). \]
Without loss of generality, When the time \( t \) satisfies
\( t_{j_k} = t_{0_k} < t_{j_{k+1}} < \ldots < t_{j_{k+m}} < t < t_{j_{k+m+1}} \), we have
\[ V(t) \leq \mu \exp(-\lambda \tau) \left[ t_{j_m+1}^m \tau_{n_{m+1}} \right] \exp(\lambda \tau(t - t_{j_k})) V(t_{j_k}). \quad (18) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq -\lambda V_{i_k, i_{k+1}} \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
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\( t_{j_k} = t_{0_k} < t_{j_{k+1}} < \ldots < t_{j_{k+m}} < t < t_{j_{k+m+1}} \), we have
\[ V(t) \leq \mu \exp(-\lambda \tau) \left[ t_{j_m+1}^m \tau_{n_{m+1}} \right] \exp(\lambda \tau(t - t_{j_k})) V(t_{j_k}). \quad (18) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq -\lambda V_{i_k, i_{k+1}} \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
Choose the piecewise function
\[ V(x(t)) = V_{i_k, i_{k+1}}(x(t)), \quad t \in [t_{j_k}, t_{j_{k+1}}). \]
Without loss of generality, When the time \( t \) satisfies
\( t_{j_k} = t_{0_k} < t_{j_{k+1}} < \ldots < t_{j_{k+m}} < t < t_{j_{k+m+1}} \), we have
\[ V(t) \leq \mu \exp(-\lambda \tau) \left[ t_{j_m+1}^m \tau_{n_{m+1}} \right] \exp(\lambda \tau(t - t_{j_k})) V(t_{j_k}). \quad (18) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq -\lambda V_{i_k, i_{k+1}} \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
Choose the piecewise function
\[ V(x(t)) = V_{i_k, i_{k+1}}(x(t)), \quad t \in [t_{j_k}, t_{j_{k+1}}). \]
Without loss of generality, When the time \( t \) satisfies
\( t_{j_k} = t_{0_k} < t_{j_{k+1}} < \ldots < t_{j_{k+m}} < t < t_{j_{k+m+1}} \), we have
\[ V(t) \leq \mu \exp(-\lambda \tau) \left[ t_{j_m+1}^m \tau_{n_{m+1}} \right] \exp(\lambda \tau(t - t_{j_k})) V(t_{j_k}). \quad (18) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq -\lambda V_{i_k, i_{k+1}} \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
Choose the piecewise function
\[ V(x(t)) = V_{i_k, i_{k+1}}(x(t)), \quad t \in [t_{j_k}, t_{j_{k+1}}). \]
Without loss of generality, When the time \( t \) satisfies
\( t_{j_k} = t_{0_k} < t_{j_{k+1}} < \ldots < t_{j_{k+m}} < t < t_{j_{k+m+1}} \), we have
\[ V(t) \leq \mu \exp(-\lambda \tau) \left[ t_{j_m+1}^m \tau_{n_{m+1}} \right] \exp(\lambda \tau(t - t_{j_k})) V(t_{j_k}). \quad (18) \]
When \( t \in [t_{j_k}, t_{j_{k+1}}) \), \( i_k \in I_p \), \( t_{j_k} = t_{j_{k-1}} + \tau(t_{j_{k-1}}) \), \( k \geq 2 \), we get \( \dot{V}_{i_k, i_{k+1}} \leq -\lambda V_{i_k, i_{k+1}} \) and
\[ V_{i_k, i_{k+1}+1}(t) \leq \exp(-\lambda(t - t_{j_k})) V_{i_k, i_{k+1}+1}(t_{j_k}). \quad (17) \]
Choose the piecewise function
\[ V(x(t)) = V_{i_k, i_{k+1}}(x(t)), \quad t \in [t_{j_k}, t_{j_{k+1}}). \]
where $A_i$, $B_i$ and $C_i$ are constant matrices of appropriate dimensions.

Combining Theorem 1, we have the following corollary immediately.

**Corollary 1** Consider the switched system (21) and let $\tau_a$, $r$ and $\tau_\Omega$ be any given average dwell time, passivity rate and any admissible switching delay satisfying $\tau_\Omega < \tau_ar$, respectively. Suppose that there exist positive definite functions $V_i = \frac{1}{2}x^TP_i x$, $V_{i,j}(x) = \frac{1}{2}x^TP_{i,j} x$, $\forall i,j \in I$ satisfying the following conditions

(i) for $i \in I_n$, there exists a constant $\lambda_i$ satisfies

$$\dot{\lambda}_i \geq \frac{\lambda_i}{\bar{T}_i}$$

for some constant $\bar{T}_i > 0$.

(ii) for $i \in I_p$, the subsystem (29) is passive, i.e.

$$\dot{\lambda}_i \leq 0$$

In addition, the passive subsystem with $u_i = 0$ is assumed to be exponentially small-time norm-observable with constants $\bar{c}$ and $c$ satisfying $\bar{c} \geq \frac{1}{2} \lambda^*$, $c \leq \frac{a_i}{a_2}$, where

$$\lambda^* = \frac{\tau_a}{r\tau_a - \tau_\Omega} - (\lambda_2 + (1-r)(\lambda_1 + \lambda_2) + 2 \ln \mu)$$

for some constant $\lambda_2 > 0$.

Further, for $\forall x \in R^n$, $\forall i,j \in I$,

$$V_{i,j}(x) \leq \mu V_{i,j}(x)$$

(22) hold. And there exists a real number $\lambda_0 > 0$, such that

$$\frac{dV_{i,j}(x)}{dt} = \frac{dV_{i,j}(x)}{dx} (f_{i}(x) + g_{i}(x) u_j) \leq \lambda_0 V_{i,j}(x)$$

hold for the controller $u_j(x) = -k_j(V_{i,j}(x))T$, $i \in I_n$, $j \in I_p$. Design controllers

$$u_i(x) \begin{cases} -k_i(L_{g_i} V_i(x))^T, & i \in I_p, \\ 0, & i \in I_n, \end{cases}$$

where $k_i = \frac{\lambda^*}{\|L_{g_i} V_i(x)\|^2} V_i(x)$, $\|L_{g_i} V_i(x)\| > \delta$.

Then the switched system (1) is exponentially stable under any switching signals with the average dwell time $\tau_a$, passivity rate $r_{\mu(\tau_a, \tau_\Omega)} \geq r$ and admissible switching delay $\tau_\Omega$.

**Remark 3.4** In [27], a method is provided to find $\mu$ in the condition (22) numerically using LMIs.

### 4 Example

In this section, we give an example to demonstrate the effectiveness of the proposed design method.

Example 1 Consider the switched nonlinear system (1) with two subsystems

$$f_1(x) = \begin{pmatrix} -3x_1^2 + x_1x_2 \\ -2x_2^2 + 3x_2 \end{pmatrix}, \quad g_1(x) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

$$h_1(x) = \begin{pmatrix} x_1^2 + x_2^2 \\ x_1^2 + x_2^2 \end{pmatrix}, \quad f_2(x) = \begin{pmatrix} x_1^2 + x_2 \\ -x_1^2 + 2x_2 \end{pmatrix}, \quad g_2(x) = \begin{pmatrix} 0 \\ x_2 \end{pmatrix}.$$
5 Conclusions

We have investigated the stabilization problem of the switched nonlinear systems consisting of passive and non-passive subsystems using an average dwell time approaches under asynchronous switching. Differently to all the existing results, for any switching law with the average dwell time, any given passivity rate, and all admissible switching delay, under the designed controllers, the stabilization problem of the switched nonlinear system is achieved.

References