Delay-Dependent Criteria of Robust Stability of Neutral Systems with Time-Varying Delay and Nonlinear Uncertainties

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Abstract: The problem of robust stability of a class of uncertain neutral systems with time-varying delay and nonlinear uncertainties is studied in this paper. The proposed criteria are both discrete-delay dependent and neutral-delay dependent. The bound of the derivative of time-varying delay is an unknown constant. As a special case, when the neutral systems degenerate to the retarded systems, the criteria obtained in this paper are less conservative than existing ones. The criteria are given in terms of linear matrix inequalities (LMIs). A simulation example illustrates the effectiveness of the developed method.

Key Words: Time-delay systems (TDS), Neutral systems, Stability, Uncertainty, Linear matrix inequality (LMI)

1. Introduction

It is well known that time-delay systems (TDS) have been an active research area for the last few decades. The main reason is that time-delay frequently occurs in many practical systems, such as mechanical transmission system, fluid transmission system, economic systems, network control system, and metallurgical industry process, and a major cause of instability and poor performance (see for example, [1]). So far, the research on the stability of TDS can be classified into two types, namely a frequency-domain approach and a time-domain approach. The time-domain approach is mainly based on the application of Lyapunov-Krasovskii Theorem and the Razumikhin Theorem. The main ideas of them are to construct a suitable Lyapunov functional or function to obtain a sufficient condition for stability [2-6]. Two types of criteria cause for concern: one is delay-independent; the other is delay-dependent. The delay-independent criteria are independent with delays, it is usually conservative for small delay systems. Therefore, delay-dependent criteria are popular.

Neutral systems as a general sense of TDS have the similar problems. Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state. For the current study of linear neutral systems by the time-domain approach, the Lyapunov-Krasovskii method is still widely used, and the stability criteria are usually proposed in terms of LMIs [7-9].

Furthermore, due to inaccuracy in model parameter measurements, data input, and any kind of unpredictability, a real system always involves uncertainties. Generally speaking, these uncertainties may give rise to instability of the system, even if the uncertainties are tiny. In the process of analysis and design for the real system, the most pressing problem is how to deal with the uncertainties.

Recently, many researchers have paid a lot of attentions on the problem of robust stability for TDS or neutral systems with nonlinear uncertainties, and many methods have been proposed to deal with the nonlinear uncertainties. In [2], some new Lyapunov-Krasovskii functionals are proposed to obtain the stability criteria which require no any constraints on the time derivative of the delays. In [3], the stability criteria are developed by descriptor model transformation and decomposition technique, and the nonlinear uncertainties are handled by S-procedure. [7] has been extended the method which is used in [3] to the neutral system with time-varying delay and nonlinear uncertainties. But the criteria which were developed in [7] is neutral delay independent, and when they estimating the upper bound of the derivative of Lyapunov functional, a useful term is ignored, which lead to considerable conservativeness.

In this paper, the problem of robust stability for uncertain neutral systems with time-varying delay and nonlinear uncertainties is studied. The bound of the derivative of time-varying delay is an unknown constant. Based on constructing a new Lyapunov functional, some robust delay-dependent stability criteria are proposed and formulated in the form of LMIs. The proposed criteria are both discrete-delay dependent and neutral-delay dependent. A numerical example is given to illustrate effectiveness of the developed technique.

This paper is organized as follows. In Section 2, we formulate the problem. Section 3 gives the main results on stability analysis of the neutral systems with time-varying discrete delay. A numerical example is given to illustrate the effectiveness of the proposed results in Section 4. Section 5 concludes the paper.

Notation. Throughout this paper, if not explicit, matrices are assumed to have compatible dimensions. $R^n$ and $R^{n×n}$
denote the $n$-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively; $A^T$ and $A^{-1}$ stand for the transpose and the inverse of a matrix $A$, respectively; $P > 0 (P < 0)$ is used to denote a symmetric positive definite (negative) matrix; $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of $P$; $\| \|$ refers to the Euclidean vector norm. * represents the elements below the main diagonal of a symmetric matrix. $I$ denotes the identity matrix of appropriate dimensions.

2. Problem Formulation

In this section, we consider a class of neutral systems with time-varying delay and nonlinear uncertainties

\[ \begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)x(t - \tau(t)) + F(t)f(x(t),t) + G(t)g(x(t - \tau(t)),t), \\
x(t) &= \psi(t), \quad t \in [- \max(\tau, h), 0],
\end{align*} \tag{1} \]

where $x(t) \in \mathbb{R}^n$ is the state vector; $\psi(t)$ is a continuous vector initial function that is continuously differentiable on $[- \max(\tau, h), 0]$, $\tau(t)$ is a time-varying delay, $h$ is a neutral delay, $C$ is a constant matrix, $f(x(t),t) \in \mathbb{R}^r$ and $g(x(t - \tau(t)),t) \in \mathbb{R}^r$ are unknown nonlinear uncertainties which with respect to the current state $x(t)$ and the delayed state $x(t - \tau(t))$. The nonlinear uncertainties are assumed to be bounded in magnitude as

\[
\left\| f(x(t),t) \right\| \leq \alpha \| x(t) \|, \\
\left\| g(x(t - \tau(t)),t) \right\| \leq \beta \| x(t - \tau(t)) \|, \quad \forall t > 0, \tag{2}
\]

where $\alpha$ and $\beta$ are known constants.

Constraint (2) can be rewritten as

\[
\begin{align*}
f^T(x(t),t)f(x(t),t) &\leq \alpha^2 x^T(t)x(t), \\
g^T(x(t - \tau(t)),t)g(x(t - \tau(t)),t) &\leq \beta^2 x^T(t - \tau(t))x(t - \tau(t)).
\end{align*} \tag{3}
\]

The uncertain matrices $A(t)$, $B(t)$, $F(t)$ and $G(t)$ satisfy

\[
\begin{align*}
A(t) &= A + \Delta A(t), \\
B(t) &= B + \Delta B(t), \\
F(t) &= F + \Delta F(t), \\
G(t) &= G + \Delta G(t),
\end{align*}
\]

where $A \in \mathbb{R}^{m \times m}$, $B \in \mathbb{R}^{n \times m}$, $F \in \mathbb{R}^{r \times n}$ and $G \in \mathbb{R}^{r \times r}$ are known constant matrices, and $\Delta A(t), \Delta B(t), \Delta F(t), \Delta G(t)$ are time-varying uncertain matrices with appropriate dimensions subject to the following forms

\[
\begin{align*}
\Delta A(t) &= D_1 F(t) E_1, \\
\Delta B(t) &= D_2 F(t) E_2, \\
\Delta F(t) &= D_3 F(t) E_3, \\
\Delta G(t) &= D_4 F(t) E_4,
\end{align*} \tag{4}
\]

where $D_i$ and $E_i$, $i = 1, 2, 3, 4$ are known constant matrices and $F(t)$ is an unknown time-varying matrix satisfies

\[ F(t)F^T(t) \leq I, \quad \forall t > 0. \tag{5} \]

It is assumed the elements of $F(t)$ are Lebesgue measurable. When $F(t) = 0$, system (1) is referred to the following nominal neutral system

\[
\begin{align*}
\dot{x}(t) &= -Cx(t - h) + Ax(t) + Bx(t - \tau(t)) + Ff(x(t),t) + Gg(x(t - \tau(t)),t), \\
x(t) &= \psi(t), \quad t \in [- \max(\tau, h), 0],
\end{align*} \tag{6}
\]

It is assumed that the time-varying delay $\tau(t)$ satisfies

\[ 0 \leq \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \dot{\tau}, \tag{7} \]

where $\tau$ and $\dot{\tau}$ are positive constants. The inequality (7) means that the bound of derivative of $\tau(t)$ can be an unknown constant.

In order to obtain the main results, some lemmas are introduced.

**Lemma 1** [10]. Let $f(x), y_1(x), y_2(x), \cdots, y_k(x)$ be some non-negative functional or functions, and define the following conditions

(a) $f(x) \geq 0$;

(b) $\varepsilon_i \geq 0, \varepsilon_1 \geq 0, \cdots, \varepsilon_k \geq 0$ such that

\[ S(\varepsilon, x) = f(x) - \sum_{i=1}^{k} \varepsilon_i y_i(x) \geq 0. \]

Then (b) implies (a).

**Lemma 2** [5]. For any dimension compatible matrices $D, E, F$ with $F^T F \leq I$, and a scalar $\varepsilon > 0$, the following inequality holds

\[
DFE + E^T F^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E. \tag{8}
\]

3. Main Results

In this section, we will perform stability analysis of uncertain neutral systems with time-varying delay and nonlinear uncertainties described by (1). We rewrite system (1) to the following equivalent descriptor system

\[
\begin{align*}
\dot{x}(t) &= y(t), \\
y(t) &= Cy(t - h) + A(t)x(t) + B(t)x(t - \tau(t)) + F(t)f(x(t),t) + G(t)g(x(t - \tau(t)),t).
\end{align*} \tag{8}
\]

Throughout this paper, we assume that

**Assumption 1.** All the eigenvalues of matrix $C$ are inside the unit circle.

Firstly, we study the problem of stability for nominal system (8). The following theorem presents a delay-dependent result in terms of LMIs.

**Theorem 1.** Under Assumption 1, the nominal system (8) is asymptotically stable if there exist real matrices $P_i, Q_i, X_{11}, X_{12}, X_{22}$ and symmetric positive definite matrices $P_i, R_i, Q_i, X_{11}, X_{12}, X_{22}, Q_i, Q_i$ and scalars $\varepsilon_i \geq 0$ and $\varepsilon_i \geq 0$ such that the following LMIs hold:
\[
\begin{bmatrix}
\hat{\Omega}_{11} & \Omega_{12} & \Omega_{13} & P^T C + M_i^T & M_i - M_i & -N_i & P^T F + M_i^T & P_i^T G + M_i^T & hM_i & \tau N_i \\
\Omega_{21} & \hat{\Omega}_{22} & P_i^T B + N_i & M_i^T - M_i & -N_i & P_i^T F & P_i^T G & hM_i & \tau N_i \\
\Omega_{31} & \hat{\Omega}_{32} & N_i^T & -M_i & -N_i & P_i^T F & P_i^T G & hM_i & \tau N_i \\
\end{bmatrix} < 0, \tag{9}
\]

where
\[
\hat{\Omega}_{11} = P_i^T A + A^T P_i + \tau X_{11} + X_{13} + X_{13}^T + \Omega_i + Q_2 + M_i + M_i^T + R_i + \alpha^2 \epsilon_i I,
\]
\[
\Omega_{21} = -P_i^T P_i^T + hR_i + R_i X_{13} + R_i^2,
\]
\[
\Omega_{31} = P_i^T + A^T P_i + M_i^T + M_i^T,
\]
\[
\hat{\Omega}_{32} = \tau X_{22} - X_{22} - X_{22}^T - (1 - \hat{\tau})Q_i + N_i^T + N_i + \beta^2 \epsilon_i I,
\]
\[
\Omega_{31} = P_i^T B + \tau X_{12} - X_{12} + X_{12}^T + M_i^T + N_i,
\]
and
\[
X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\
* & X_{22} & X_{23} \end{bmatrix} = X^T > 0. \tag{10}
\]

**Proof.** Constructing a Lyapunov–Krasovskii functional as follows
\[
V(t) = \sum_{i=1}^5 V_i(t), \tag{11}
\]
where
\[
V_1(t) = x^T(t)Px(t), \tag{12}
\]
\[
V_2(t) = \int_0^t \int \dot{x}(s)R_1\dot{x}(s)ds + \int \dot{x}(s)R_2\dot{x}(s)ds + \int x(s)R_3x(s)ds, \tag{13}
\]
\[
V_3(t) = \int_0^t (\tau - s)x^T(t - s)X_{22}^T x(t - s)ds, \tag{14}
\]
\[
V_4(t) = \int_0^t \int \epsilon^T Xeddsd\sigma, \tag{15}
\]
with \(e = \begin{bmatrix} x(\sigma) \\
\dot{x}(\sigma - \tau(\sigma)) \end{bmatrix}\),
\[
\dot{x}(\sigma)
\]
and
\[
X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\
* & X_{22} & X_{23} \end{bmatrix} = X^T > 0,
\]
\[
V_5(t) = \int_{[\tau - \tau(t)]} x(s)Q_1x(s)ds + \int_{[\tau - \tau(t)]} x(s)Q_2x(s)ds. \tag{16}
\]

From the Leibniz-Newton formula
\[
\int_{t - \tau}^{t} \dot{x}(s)ds = x(t) - x(t - \tau)
\]
for matrices \(M^T = [M_1^T, \ldots, M_i^T], N^T = [N_1^T, \ldots, N_i^T] \) with arbitrary appropriate dimensions, we have
\[
\alpha = 2\zeta^T(t)M[x(t) - x(t - h)] - \int_{t - h}^t \dot{x}(s)ds = 0,
\]
\[
\beta = 2\zeta^T(t)N[x(t - \tau(t)) - x(t - \tau)] - \int_{t - \tau(t)}^{t - \tau(t)} \dot{x}(s)ds = 0,
\]
where
\[
\zeta(t) = [x^T(t) \ y^T(t) \ x^T(t - \tau(t)) \ y^T(t - h) \ x^T(t - h)]
\]
Now, we consider the time derivative of \(V(t) \) along the solution trajectories of system (8).
\[
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_4(t) + \dot{V}_5(t) + \dot{V}_6(t) + \alpha + \beta. \tag{17}
\]
For the first term \(V_1(t)\), we have \(\forall P_i, P_i^T\)
\[
\dot{V}_1(t) = 2x^T(t)P_i\dot{x}(t) + 2x^T(t)P_i^T y(t)
\]
\[
= 2x^T(t)P_i\dot{x}(t) + 2x^T(t)P_i^T y(t) \tag{18}
\]
\[
\leq 2x^T(t)P_i\dot{x}(t) + 2x^T(t)P_i^T y(t) + 2x^T(t)P_i^T Cy(t - h)
\]
\[
+ 2x^T(t)P_i^T Ax(t) + 2x^T(t)P_i^T Bx(t - \tau(t)))
\]
\[
+ 2x^T(t)P_i^T Ff(x(t),t) + 2x^T(t)P_i^T Gg(x(t - \tau(t)),t)
\]
\[
- 2y^T(t)P_i^T y(t) + 2y^T(t)P_i^T Cy(t - h) + 2y^T(t)P_i^T Ax(t)
\]
\[
+ 2y^T(t)P_i^T Bx(t - \tau(t)) + 2y^T(t)P_i^T Ff(x(t),t)
\]
\[
+ 2y^T(t)P_i^T Gg(x(t - \tau(t)),t).
\]

The derivative of \(V_2(t)\) is
\[
\dot{V}_2(t) = h\dot{y}^T(t)R_iy(t) - \int_{t - \tau(t)}^t \dot{y}^T(s)R_iy(s)ds
\]
\[
+ y^T(t)R_iy(t) - y^T(t - h)R_iy(t - h)
\]
\[
+ x^T(t)R_i(x(t) - x(t - h))R_iy(t - h).
\]
Equation (14) can be described as:
\[
V_5(t) = \int_{t - \tau}^{t} (x + \tau - s)\dot{x}^T(s)X_{22}\dot{x}(s)ds
\]
Finally, we have

\[
V_3(t) = \tau \dot{\xi}^T(t)X_{33}\dot{\xi}(t) - \int_{t-\tau}^t \dot{\xi}^T(s)X_{33}\dot{\xi}(s)ds
\]

The derivative of \(V_3(t)\) is

\[
\dot{V}_3(t) = \tau \dot{\xi}^T(t)X_{33}\dot{\xi}(t) - \int_{t-\tau}^t \dot{\xi}^T(s)X_{33}\dot{\xi}(s)ds
\]

It is obviously that

\[
\dot{V}_3(t) = \xi^T(t)Q_4\xi(t) - (1 - \tilde{\eta})\xi^T(t - \tau(t))Q_4\xi(t - \tau(t)) + \xi^T(t)Q_4\xi(t) - \dot{\xi}^T(t - \tau)\xi(t - \tau).
\]

Finally, we have

\[
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) + \alpha + \beta
\]

\[
\leq \xi^T(t)[\Omega + hMR_1^{-1}M^T + \tau NX_3^{-1}N^T]\xi(t)
\]

\[
- \int_{t-\tau}^t \xi^T(t)M + \xi^T(s)R_1R_1^{-1}[M^T\xi(t) + R_1\dot{\xi}(s)]ds
\]

\[
- \int_{t-\tau}^t \xi^T(t)N + \xi^T(s)X_{33}^1[N^T\xi(t) + X_{33}\dot{\xi}(s)]ds,
\]

where

\[
\Omega =
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & P^T_1C + M^T_1 & M^T_1 - M_1 \\
\Omega_{21} & \Omega_{22} & \Omega_{23} & P^T_2C & -M_2 \\
\Omega_{31} & \Omega_{32} & \Omega_{33} & N_4^T & N_4^T - M_3 \\
* & * & * & -R_2 & -M_4 \\
* & * & * & * & \Omega_{44} \\
* & * & * & * & * \\
* & * & * & * & *
\end{bmatrix}
\]

\[
\begin{bmatrix}
-N_1 & P^T_1F + M^T_1 & P^T_1G + M^T_1 \\
-N_2 & P^T_2F & P^T_2G \\
N_4^T - N_3 & N_4^T & N_4^T \\
-N_5 & 0 & 0 \\
-M_2^T & -M_2^T & \Omega_{55} - N_5^T & -N_5^T \\
* & 0 & 0 & * & 0
\end{bmatrix}
\]

\[
\Omega_1 = P^T_1A + A^T_1P + \tau X_{11} + X_{11}^T + Q_1 + Q_2 + M_1 + M_1^T + R_1,
\]

\[
\Omega_{21} = -P_2^T - P_2 + hR_2 + \tau X_{33} + R_2,
\]

\[
\Omega_{22} = -P_2^T - P_2 + hR_2 + \tau X_{33} + R_2,
\]

\[
\Omega_{31} = \tau X_{22} - X_{22} + X_{22}^T - (1 - \tilde{\eta})Q_4 + N_4^T + N_4^T,
\]

\[
\Omega_{32} = P_1^T + \tau X_{12} - X_{12} + X_{12}^T + M_1 + M_1^T + N_4^T + N_4^T,
\]

\[
\Omega_{33} = P_2^T + N_4^T + N_4^T,
\]

\[
\Omega_{44} = -R_2 - M_4 - M_4^T,
\]

\[
\Omega_{55} = -Q_4 - N_5^T - N_5^T.
\]

The condition \(R_1, X_{33} > 0\) implies

\[
\dot{V}(t) \leq \zeta^T(t)[\Omega + hMR_1^{-1}M^T + \tau NX_3^{-1}N^T]\zeta(t)
\]

Obviously, nominal system (8) is asymptotically stable if \(\Omega + hMR_1^{-1}M^T + \tau NX_3^{-1}N^T < 0\) is satisfied for all \(\zeta(t) \neq 0\). Based on Lemma 1, we know that this condition is implied by the existence of nonnegative scalars \(c_i \geq 0, i = 0, 1, \ldots, 6\) such that

\[
\zeta^T(t)[\Omega + hMR_1^{-1}M^T + \tau NX_3^{-1}N^T]\zeta(t) + \varepsilon_1[\alpha^T\xi^T(t)\xi(t)]
\]

\[
- f^T(x(t), t)f(x(t), t)] + \varepsilon_2[\beta^T\xi^T(t - \tau(t))\xi(t - \tau(t))]
\]

\[
- g^T(x(t - \tau(t)), t)g(x(t - \tau(t)), t) < 0.
\]

Using Schur complement theorem, inequality (19) is equivalent to inequality (9). Inequalities (9) implies \(\dot{V}(t) \leq -\gamma\|x(t)\|_2^2\), where \(\gamma\) is a sufficiently small positive scalar. Therefore, nominal system (8) is asymptotically stable.

Based on Theorem 1, we can perform the robust stability analysis for system (1) with uncertainties (3) and (4).

**Theorem 2.** Under Assumption 1, system (1) with uncertainties (3) and (4) is asymptotically stable if there exist real matrices \(P_1, P_2, X_{11}, X_{12}, X_{22}, X_{33}, Q_1, Q_2\) and symmetric positive definite matrices \(P, R_1, R_2, R_3, X_{11}, X_{22}, X_{33}, Q_1, Q_2\) and scalars \(\varepsilon_i > 0, i = 1, \ldots, 6\), such that the following LMIs hold:
where

\[ \Omega_{11} = P_1^T A + A^T P_1 + r X_{11} + X_{11} + X_{13} + Q_1 + Q_3 + M_i + M_i^T R_1 + c_2^2 \varepsilon_i I + \varepsilon_i E_i^T E_i , \]
\[ \Omega_{22} = -P_2^T - P_2 + h R_1 + \tau X_{33} + R_2 , \]
\[ \Omega_{31} = P_1^T A + A^T P_1 + M_i^T , \]
\[ \Omega_{32} = -P_2^T - P_2 + h R_1 + \tau X_{33} + R_2 , \]
\[ \Omega_{33} = P_1^T B + \tau X_{13} + X_{13} + X_{33} + M_i^T + N_i , \]
\[ \Omega_{41} = P_2^T B + N_2 , \]
\[ \Omega_{44} = -R_3 - M_i^T , \]
\[ \Omega_{42} = -Q_2 - N_6 - N_6^T , \]
\[ \Omega_{33}' = -\tau X_{33} - X_{33}^T - (1 - \tau) Q_1 + N_i^T \]
\[ + N_1 + P_1^T A + A^T P_1 + \varepsilon_i E_i^T E_i , \]
\[ \Omega_{32}' = P_2^T B + N_2 , \]

and

\[ X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{22} & X_{23} \\ * & * & \end{bmatrix} = X^T > 0 . \]

**Proof.** Replacing \( A, B, F \) and \( G \) in (9) with \( A + D_1 F(t) E_1, B + D_2 F(t) E_2, F + D_3 F(t) E_3 \) and \( G + D_4 F(t) E_4 \), respectively. Based on Lemma 2, the desired result can be carried out by multiplying the left and the right sides of (20) by vector \( \xi^T \) and \( \xi \), respectively, together with Theorem 1, where

\[ \xi^T(t) = [x^T(t) \ y^T(t) \ x^T(t-h) \ y^T(t-h) \ x^T(t-h) \ y^T(t-h)] \]
\[ x^T(t - \tau) \ f^T(x(t),t) \ g^T(x(t - \tau(t)),t) \]
\[ x^T(t) E_i^T F_i^T \ x^T(t - \tau(t)) E_i^T F_i^T \]
\[ f^T(x(t),t) E_i^T F_i^T \ g^T(x(t - \tau(t)),t) E_i^T F_i^T(t) \].

**Remark.** We improve the existing results due to the method to estimate the upper bound of the derivative of Lyapunov functional without ignoring the useful term, thus reducing the conservatism. Moreover, the bound of derivative of the time-varying delay is an unknown constant and the proposed stability criteria are both discrete and neutral delays dependent, which further reduce the conservative.

### 4. Numerical Example

In this section, an example is presented to illustrate the effectiveness of the main results.

**Example.** Consider the following uncertain neutral system with time-varying delay and nonlinear uncertainties

\[ \dot{x}(t) - C \dot{x}(t - h) = (A + A(t))x(t) + (B + B(t))x(t - \tau(t)) \]
\[ + (F + F(t)) f(x(t),t) + (G + G(t)) g(x(t - \tau(t),t)), \tag{21} \]

where

\[ A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \]
\[ F = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, G = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}. \]

and

\[ \Delta A(t) = D_1 F(t) E_1, \Delta B(t) = D_2 F(t) E_2, \]
\[ \Delta F(t) = D_3 F(t) E_3, \Delta G(t) = D_4 F(t) E_4, \]
\[ F^T(t) F(t) \leq I, D_i = \gamma I, E_i = I, i = 1,2,3,4. \]

It is assumed that the nonlinear uncertainties satisfy

\[ \| f(x(t),t) \| \leq \alpha \| x(t) \|, \]
\[ \| g(x(t - \tau(t),t)) \| \leq \beta \| x(t - \tau(t)) \|, \quad \alpha > 0, \beta > 0. \]

Applying the criteria in [7] and our criteria, the maximum \( \tau_{\text{max}} \) of \( \tau \) is list in the following tables 1 and 2 for different values of \( \gamma \). We have the comparative results listed in Table 1 and 2. From the comparative results, we can see...
that our results are much less conservative than those in [7]. Moreover, the stability criteria in ours’ can be applied to the case that the derivative of the discrete delay is more than 1.

Table 1

<table>
<thead>
<tr>
<th>h = 0.1</th>
<th>α = 0, β = 0.1</th>
<th>α = 0, β = 0.1</th>
<th>α = 0, β = 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ = 0</td>
<td>1.180</td>
<td>0.704</td>
<td>----</td>
</tr>
<tr>
<td>τ = 0.5</td>
<td>1.8608</td>
<td>0.9167</td>
<td>0.6371</td>
</tr>
<tr>
<td>τ = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

max of [7] 1.180 0.704 ----
max of our results 1.8608 0.9167 0.6371

If we let \( F(t) = 0, F = G = I \) and \( C = 0 \), the system (21) is reduced to the system considered in [3] and [4], then applying the criteria in [3] [4] and our criteria, we have the following comparative results listed in Table 3. From Table 3, we can see that our results are much less conservative than those in [3, 4].

Table 2

<table>
<thead>
<tr>
<th>h = 0.1</th>
<th>α = 0, β = 0.1</th>
<th>α = 0, β = 0.1</th>
<th>α = 0, β = 0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ = 0</td>
<td>0.306</td>
<td>0.259</td>
<td>----</td>
</tr>
<tr>
<td>τ = 0.5</td>
<td>0.5409</td>
<td>0.3685</td>
<td>0.4312</td>
</tr>
<tr>
<td>τ = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

max of [7] 0.306 0.259 ----
max of our results 0.5409 0.3685 0.4312

5. Conclusions

In this paper, the problem of robust stability of neutral systems with time-varying delay and norm bounded uncertainties is considered. The bound of derivative of the time-varying delay can be an unknown constant. Sufficient conditions of stability are both discrete-delay dependent and neutral-delay dependent, and are given in terms of LMIs. Numerical example is given to illustrate the advantages of the theoretic results obtained, and show that our results are much less conservative than some existing results in the literatures.

References