Switching Control for LPV Polytopic Systems Using Multiple Lyapunov Functions

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Abstract: In this paper, we study the switching control for a linear parameter varying (LPV) polytopic system using multiple Lyapunov functions to improve the system’s $H_{\infty}$ performance. For the large range of parameter varying, we divide the parameter region into small subregions and find a suitable Lyapunov function for each parameter subregion. Under the average dwell time switching logic, a sufficient Linear Matrix Inequality (LMI) condition is proposed to guarantee the performance. The proposed control scheme is applied to an active magnetic bearing system.

Key Words: LPV Systems, Polytopic Parameters, Multiple Lyapunov Functions, LMIs

1 Introduction

LPV control theory provides a systematic gain-scheduling design technique [1–6]. LPV systems can be viewed as linear time-invariant (LTI) plants subject to time-varying parameter and they can be models of linear time-varying plants or result from the linearization of nonlinear plants along trajectories of parameters. If parameters of systems varies in a polytope, an LPV system can be described as an LPV polytopic system. An Lyapunov function of an LPV polytopic system can be obtained by LMI conditions [6]. Gain scheduling for an LPV polytopic system is an answer for practical situations where real-time measurements are available to tune the controller according to parameter variations. Such situations are commonly encountered in a number of real-world problems, and this modeling and control method are applied in many fields such as missiles [1], aircrafts [7], energy production systems [8–10].

Closely related to LPV systems, switched systems can be described by an interaction between continuous time systems and discrete switching events, which usually depend on states or time [11]. Stability analysis of such switched systems is an important and challenging problem, and has received considerable attention in the recent literature [11, 12]. For a family of stable LTI systems, if there exists a common Lyapunov function, the stability can be guaranteed under arbitrary switchings [11]. However, this type of stability condition is deemed to be too conservative. The multiple Lyapunov functions method in switching systems have been shown to be a very useful tool for stability analysis. In the multiple Lyapunov functions method, one or more Lyapunov functions are designed for each individual subsystems being switched. Stability properties of switched systems in general depend on underlying switching logic, which is a rule that determines the switching between subsystems. Various switching rules have been proposed for the stabilization of diverse switched systems.

On the other hand, dwell time is one of switching logics which guarantee the system stability. If each subsystem is asymptotically stable, we can restrict the switching signals dwell in a system for enough long time to guarantee the stability of the system. The average dwell time relaxes the dwell time method by allowing the possibility of switching fast when necessary and then compensating for it by switching sufficiently slowly later [11]. The results of switched LTI systems have been generalized to the analysis and control of switched LPV systems [13], which is further extended in [14] by introducing average dwell time switching logic [15].

Recently, researchers introduced switching strategy into LPV systems to deal with the practical system or improve the system performance [11, 16, 17]. But results on the design of switching strategy for LPV polytopic systems are few. If we can get the vertices of a polytope of parameter region, controllers can be easily designed for the LPV polytopic system. In this research, we will study the switching control design of LPV polytopic systems using multiple Lyapunov functions. The goal of our research is to obtain the better $H_{\infty}$ performance level through dividing the large parameter region into small subregions. The main motivation of our work lies in [17, 18]. These apply the region division method to LPV systems using multiple parameter dependent Lyapunov functions with parameter region gridding method. Considering the own characteristic of polytope, we apply the division method to the LPV polytopic systems with focusing on the vertices more than gridding point, which relieves the burden of LMIs’ solving. For an LPV polytopic system, the single Lyapunov function design method needs the Lyapunov function satisfies some condition which are satisfied in whole parameter varying region. That is, in LPV polytopic systems, the LMI condition has to be satisfied for all vertices of polytope of the parameter region [1]. If parameters span of a large range, the conservatism will increase or even the LMIs are unsolvable. The division which we apply to the large polytopic region is to reduce conservatism. With the region divided, an LPV polytopic system is transformed into switched LPV polytopic system. We obtain different Lyapunov functions in each divided parameter varying subregion, and we also introduce the average dwell time switching
logic to guarantee the stability of the system with parameter 
varying in whole parameter region.

The paper is organized as follows: Section 2 provides a 
brief introduction of a switched LPV polytopic system. In 
section 3, we design multiple Lyapunov functions for an 
LPV polytopic system with average dwell time. In section 
4, an example is used to demonstrate the proposed switching 
LPV control techniques. Finally, conclusions are in section 
5.

The notation is standard in this paper. \( \mathbb{R} \) stands for the 
set of real numbers. \( \mathbb{R}^{n \times n} \) is the set of real \( m \times n \) ma-
trices. \( I \) denotes the identity matrix with compatible di-
mension. \( \text{Ker}(M) \) denote the orthogonal complement of 
\( M \). \( \mathbb{S}^{n \times n} \) is to denote the real symmetric matrices and 
\( \mathbb{S}_+^{n \times n} \) to denote positive definite matrices. If \( M \in \mathbb{S}^{n \times n} \), then \( M \succ 0 \) denotes a negative definite (negative 
semidefinite) matrix. The "?" in matrices denotes the ele-
ments we do not care.

2 Problem Statement and Preliminaries

Consider an LPV plant governed by the equation

\[
\begin{bmatrix}
\dot{x} \\
\dot{z} \\
y
\end{bmatrix} = \begin{bmatrix}
A(\rho) & B_1(\rho) & B_2(\rho) \\
C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\
C_2(\rho) & D_{21}(\rho) & 0
\end{bmatrix} \begin{bmatrix}
x \\
w \\
u
\end{bmatrix},
\]

(1)

where \( x \in \mathbb{R}^n \) are states, \( z \in \mathbb{R}^{n_z} \) are controlled outputs, and 
\( w \in \mathbb{R}^{n_w} \) are disturbance inputs. \( y \in \mathbb{R}^{n_y} \) are measured out-
puts for control, and \( u \in \mathbb{R}^{n_u} \) are control inputs, \( \rho \) are vary-
ving parameters. The state-space matrices depend affinely on 
time-varying parameters \( \rho \), and the measurements of \( \rho \) are 
available in real time. The plant is further assumed to be 
(polytopic. That is,

- The matrices \( A(\rho), B_i(\rho), C_i(\rho), D_{ij}(\rho) \), \( i,j = 1, 2 \)
depend affinely on \( \rho \);
- The time-varying parameter \( \rho \) varies in a polytope \( \mathcal{P} \) of 
vertices \( \omega_1, \omega_2, \cdots, \omega_r \), that is,

\[
\rho \in \mathcal{P} := \text{Co}(\omega_1, \omega_2, \cdots, \omega_r) = \left\{ \sum_{i=1}^r \alpha_i N_i : \alpha_i \geq 0, \sum_{i=1}^r \alpha_i = 1 \right\}
\]

(2)

Then, the LPV polytopic system can be described as

\[
\begin{bmatrix}
A(\rho) & B_1(\rho) & B_2(\rho) \\
C_1(\rho) & D_{11}(\rho) & D_{12}(\rho) \\
C_2(\rho) & D_{21}(\rho) & D_{22}(\rho)
\end{bmatrix} \in \mathcal{P}
\]

(3)

where \( A_l, B_{il}, C_{il}, D_{il;j} \) denote the values of \( A(\rho), B_i(\rho), C_i(\rho), D_{ij}(\rho) \) at the vertices \( \rho = \omega_l \) of the 
parameter polytope. We also assume that

1) \((A(\rho), B_2(\rho)), (A(\rho), C_2(\rho))\) are stabilizable and 
detectable over \( \mathcal{P} \) respectively;
2) The matrix functions \( \{B_1^T(\rho) \ D_{12}^T(\rho) \} \) and 
\( \{C_2(\rho) \ D_{21}(\rho) \} \) have full row ranks for all \( \rho \).

From the Lemma 1 in [1], the LMIs (10) have to be satis-
fied for all the vertices of the parameter polytope. If the pa-
parameter varies in a large range, that is the vertices of polytope 
span of large space, the \( \mathcal{H}_\infty \) performance \( \gamma \) will increase.
We can conclude that for this condition, the conservatism 
increases.

In our study, for the large range of parameter varying, we 
divide the large parameter polytope into small ones. We 
introduce the switching strategy in the controller design to 
make the \( \mathcal{H}_\infty \) performance \( \gamma \) less conservative.

Suppose that the parameter region \( \mathcal{P} \) is covered by a 
finite number of closed subregions \( \{\mathcal{P}_i\}_{i=1}^{Z_N} \) by means of 
a family of switching surfaces, where the index set \( Z_N = 
\{1, 2, \cdots, N\} \), and \( \mathcal{P} = \bigcup \mathcal{P}_i \). Each divided subregion 
\( \mathcal{P}_i \) must be a polytope. The adjacent parameter subsets are 
separated by switching surfaces, and they have disjointed in-
teriors. We are interested in the problem of design of 

- designing a family of LPV controllers in the form of

\[
\begin{bmatrix}
\dot{x}_k \\
u
\end{bmatrix} = \begin{bmatrix}
A_{k,i}(\rho) & B_{k,i}(\rho) \\
C_{k,i}(\rho) & D_{k,i}(\rho)
\end{bmatrix} \begin{bmatrix}
x_k \\
y
\end{bmatrix}, \quad i \in \{1, 2, \cdots, N\}
\]

(4)

Under switching LPV control, the closed-loop LPV sys-

tem can be described by

\[
\begin{bmatrix}
\dot{x}_{cl} \\
u
\end{bmatrix} = \begin{bmatrix}
A_{cl,\sigma}(\rho) & B_{cl,\sigma}(\rho) \\
C_{cl,\sigma}(\rho) & D_{cl,\sigma}(\rho)
\end{bmatrix} \begin{bmatrix}
x_{cl} \\
w
\end{bmatrix},
\]

(5)

where \( x_{cl} = [x^T \ x_k^T] \in \mathbb{R}^{n_{cl}+n_k} \). It is straightforward to 
show that the resulting closed-loop system is a switched LPV 
system. The switching signal \( \sigma \) is defined as a piecewise 
function. It is assumed that \( \sigma \) is continuous from the right 
wherever and only a limited number of switchings happen in 
any finite time interval.

To develop our main result, we need the following lemma.
We introduce the Bounded Real Lemma state-space (BRL) 
map \( B_{(A,B,C,D)}(X, \gamma) \) defined for symmetric matrices \( X \) and 
positive scalar \( \gamma \) by:

\[
B_{(A,B,C,D)}(X, \gamma) := \begin{bmatrix}
ATX + XA & XB \\
BTX & -\gamma I
\end{bmatrix}.
\]

(6)

Then we have the following lemma in [1].

**Lemma 1** ([1]) Consider a polytopic LPV plant described by the state-space equations

\[
\dot{x} = A(\rho)x + B(\rho)u, \\
y = C(\rho)x + D(\rho)u,
\]

(7)

with

\[
\begin{bmatrix}
A(\rho) & B(\rho) \\
C(\rho) & D(\rho)
\end{bmatrix} \in \mathcal{P} := \text{Co}\left\{ \begin{bmatrix}
A_l & B_l \\
C_l & D_l
\end{bmatrix} : l = 1, \cdots, r \right\}.
\]

(8)

The following statements are equivalent:

1) this LPV system is stable with quadratic \( \mathcal{H}_\infty \) perfor-

mance \( \gamma \):

2) there exists a single matrix \( X > 0 \) such that, for all

\[
\begin{bmatrix}
A(\rho) & B(\rho) \\
C(\rho) & D(\rho)
\end{bmatrix} \in \mathcal{P},
\]

(9)

\[
B_{(A(\rho),B(\rho),C(\rho),D(\rho))}(X, \gamma) < 0;
\]

3) there exists \( X > 0 \) satisfying the system of LMIs

\[
B_{(A_l,B_l,C_l,D_l)}(X, \gamma) < 0, l = 1, 2, \cdots, r.
\]

(10)
3 Main Result

Theorem 1 Given scalars $\lambda_0 > 0$, $\mu > 1$, an open-loop LPV system (1), the parameter set $\mathcal{P}$ and its partition $\{\mathcal{P}_i\}_{i=1}^{2N}$, Assume one of the following conditions are satisfied:

(1) there exist positive definite matrices $R_i, S_i$: $R^* \to S_i^{n \times n}$, $i \in Z_N$, such that for any $i \in \mathcal{P}_i$, 

\[
N_i^T \left( \begin{array}{c} R_i \ 1 \ S_i \end{array} \right) \geq 0 \quad (13)
\]

where $R_i := R_i A_i^T + A_i (\rho) R_i + \lambda_0 R_i, S_i := A_i^T (\rho) S_i + S_i A_i (\rho) + \lambda_0 S_i, N_R = \text{Ker} \left( B_i \frac{D_i^T \left( \begin{array}{c} 0 \\ 1 \\ S_i \end{array} \right)}{D_i^T (\rho)} \right)$ and $N_S = \text{Ker} \left( C_i \frac{D_i^T \left( \begin{array}{c} 0 \\ 1 \\ S_i \end{array} \right)}{D_i^T (\rho)} \right)$, and for any $\rho \in \mathcal{P}_i$, 

\[
\frac{1}{\mu} R_i \leq R_i \leq \mu R_i, \quad (14)
\]

\[
\frac{1}{\mu} (S_i - R_i^{-1}) \leq S_i - R_i^{-1} \leq \mu (S_i - R_i^{-1}) \quad (15)
\]

(2) the inequalities (11)-(13) hold and 

\[
\frac{1}{\mu} S_j \leq S_i \leq \mu S_j, \quad (16)
\]

\[
\frac{1}{\mu} (R_j - S_i^{-1}) \leq R_i - S_i^{-1} \leq \mu (R_j - S_i^{-1}) \quad (17)
\]

Then the closed-loop LPV system (1) is exponentially stabilized by a switched LPV controller over the entire parameter set $\mathcal{P}$ for every switching signal $\sigma$ with average dwell time 

\[
\tau_a > \ln \frac{\mu}{\lambda_0}, \quad (18)
\]

and for $\lambda < \lambda_0$ the system achieves a weighted disturbance attenuation level $\gamma$ in the sense of $\int_0^\infty e^{-\lambda \tau} z^T (\tau) z (\tau) d\tau \leq V_\sigma (0) + \gamma \int_0^\infty w^T (\tau) w (\tau) d\tau$ with $\gamma = \max \{ \{ \gamma_i \} \}_{i=1}^{2N}$.

Proof Without loss of generality, we only give the proof when the condition (1) holds, and the proof process condition (2) is similar. According to [5], the LMI conditions (11)-(13) are equivalent to 

\[
\begin{bmatrix}
X_i \\
B_{cl,i}^T (\rho) X_i \\
\frac{D_{cl,i}^T (\rho)}{C_{cl,i} (\rho)}
\end{bmatrix}
\begin{bmatrix}
X_i \\
B_{cl,i} (\rho) \\
\frac{D_{cl,i} (\rho)}{C_{cl,i} (\rho)}
\end{bmatrix}
\leq 0, \quad (19)
\]

and the Lyapunov function matrix of the closed-loop system can be partitioned as 

\[
X_i = \begin{bmatrix}
S_i & N_i \\
N_i^T & ?
\end{bmatrix}, \quad X_i^{-1} = \begin{bmatrix}
R_i & M_i \\
M_i^T & ?
\end{bmatrix},
\]

where $X_i := A_{cl,i}^T (\rho) X_i + X_i A_{cl,i} (\rho) + \lambda_0 X_i, M_i N_i^T = I - R_i S_i$.

We choose $M_i = R_i$ and $N_i = R_i^{-1} - S_i$. Then, the Lyapunov function can be chosen as 

\[
X_i = \begin{bmatrix}
S_i & R_i^{-1} - S_i \\
R_i^{-1} - S_i & S_i - R_i^{-1}
\end{bmatrix},
\]

and each $X_i$ can be decomposed as 

\[
X_i = \begin{bmatrix}
I - I & R_i^{-1} \\
0 & 0
\end{bmatrix} \begin{bmatrix}
S_i - R_i^{-1} & -I \\
0 & I
\end{bmatrix} - \begin{bmatrix}
0 & I
\end{bmatrix} \begin{bmatrix}
I - I
\end{bmatrix}.
\quad (20)
\]

Then the constraints (14)-(15) are equivalent to $\frac{1}{\mu} V_i \leq V_i \leq \mu V_i$.

From (19), we have 

\[
A_{cl,i}^T (\rho) X_i + X_i A_{cl,i} (\rho) \leq -\lambda_0 X_i,
\]

so the $V_i = x_i^T X_i x_i$ is a Lyapunov function of system (5) for the time-varying parameter $\rho \in \mathcal{P}_i$. We assume the sequence of finite switching time over the interval $[0, T]$ is $t_0, t_1, \ldots, t_{N_e} (0, T)$ with $t_0 = 0$. Then we have 

\[
V_\sigma (x_{cl}) \leq e^{-\lambda_0 (t_k - t_{k-1})} V_\sigma (x_{cl}(t_{k-1})) \leq \mu e^{-\lambda_0 (t_k - t_{k-1})} e^{-\lambda_0 (t_k - t_{k-1})} V_\sigma (x_{cl}(t_{k-1})) \leq e^{-\lambda_0 (t_k - t_{k-1})} V_\sigma (x_{cl}(t_0)) = e^{-\lambda_0 (t_k - t_{k-1})} V_\sigma (x_{cl}(t_0)),
\]

where $N_e (\tau, t)$ denotes the number of switchings of $\sigma$ over the interval $(\tau, t)$. The exponential stability of the closed-loop system (5) is achieved.

We can deduce from (19) that 

\[
\begin{bmatrix}
C_{cl,i} (\rho) \\
D_{cl,i} (\rho)
\end{bmatrix}
\begin{bmatrix}
X_i \\
X_i B_{cl,i} (\rho)
\end{bmatrix}
\leq 0, \quad (22)
\]

for any $\rho \in \mathcal{P}_i$.

Consider the quadratic form 

\[
\begin{bmatrix}
x \\
w
\end{bmatrix}

\begin{bmatrix}
C_{cl,i} (\rho) \\
D_{cl,i} (\rho)
\end{bmatrix}
\begin{bmatrix}
x \\
w
\end{bmatrix}
\leq 0
\]

Let $\gamma = \max \{ \{ \gamma_i \} \}_{i=1}^{2N}$, according to (22) and (23), then we have 

\[
V_i \leq e^{-\lambda_0 t} V_i - \gamma w^T w + \lambda_0 \gamma_1 V_i.
\]

From (14)-(15) and the conclusion in [19], we have 

\[
\int_0^\infty e^{-\lambda \tau} z^T (\tau) z (\tau) d\tau \leq V_\sigma (0) + \gamma \int_0^\infty w^T (\tau) w (\tau) d\tau.
\]

Motivated by the method in [19]. We calculate the derivative of $e^{\lambda_0 t} V_\sigma$ to get 

\[
e^{\lambda_0 t} \left( \lambda_0 V_\sigma + \dot{V}_\sigma \right) \leq e^{\lambda_0 t} (z^T z - \gamma w^T w),
\]

(26)
Eliminating $e^{\lambda t}$ and integrating both sides of (26), gives

$$
V_\sigma (t) \leq e^{-\lambda t} V_\sigma (0) - \int_0^t e^{-\lambda (t-\tau)} N_\sigma (t) \ln \mu V_\sigma (\tau) d\tau,
$$

(27)

where $\Gamma (\tau) := z^T (\tau) z (\tau) - \gamma^2 w^T (\tau) w (\tau)$. We multiply both sides of (27) by $e^{-N_\sigma (0, t) \ln \mu}$ to get

$$
e^{-N_\sigma (0, t) \ln \mu} V_\sigma (t) \leq e^{-\lambda t} V_\sigma (0) - \int_0^t e^{-\lambda (t-\tau)} N_\sigma (0, \tau) \ln \mu \Gamma (\tau) d\tau,
$$

(28)

which is equivalent to

$$
e^{-N_\sigma (0, t) \ln \mu} V_\sigma (t) + \int_0^t \mathcal{E} z^T (\tau) z (\tau) d\tau \leq e^{-\lambda t} V_\sigma (0) - \int_0^t \mathcal{E} w^T (\tau) w (\tau) d\tau,
$$

(29)

where $\mathcal{E} := e^{-\lambda (t-\tau)} N_\sigma (0, \tau) \ln \mu$.

Now, we choose a positive scalar $\lambda$ smaller than $\lambda_0$ to consider the following average dwell time scheme: for any $\tau > 0$,

$$
N_\sigma (0, \tau) \leq \frac{\tau}{\tau_{a}}, \quad \tau_a = \frac{\ln \mu}{\lambda},
$$

(30)

We know that $N_\sigma (0, \tau) \ln \mu \leq \lambda \tau$ holds for any $\tau > 0$. Then, from (29) and $V (t) \geq 0$, we obtain

$$
\int_0^t e^{-\lambda (t-\tau)} - \lambda \tau z^T (\tau) z (\tau) d\tau \leq e^{-\lambda t} V_\sigma (0) + \gamma^2 \int_0^t e^{-\lambda (t-\tau)} w^T (\tau) w (\tau) d\tau.
$$

(31)

Integrating (31) from $t = 0$ to $\infty$, we have

$$
\frac{1}{\lambda_0} \int_0^\infty e^{-\lambda_0 \tau} z^T (\tau) z (\tau) d\tau \leq \frac{1}{\lambda_0} V_\sigma (0) + \gamma^2 \int_0^\infty w^T (\tau) w (\tau) d\tau,
$$

(32)

and thus

$$
\int_0^\infty e^{-\lambda_0 \tau} z^T (\tau) z (\tau) d\tau \leq V_\sigma (0) + \gamma^2 \int_0^\infty w^T (\tau) w (\tau) d\tau
$$

holds for any $w (t) \in \mathcal{L}_2 [0, \infty)$. Then we finish the proof.

\begin{remark}
In LMI{s} of the Theorem 1, the number of LMI{s} with varying parameter $\rho$ are infinite. According to the results of [1] and [5], this condition can be reduced to a finite set of LMI{s} in the case of LP polytopic systems.

The term $R_{11}^{-1}$, appears in (15), which is difficult to solve. We take the similar constraint in [18] that we enforce $R_{11} = R_f$. Then the LMI{s} can be easily solved.
\end{remark}

\section{Example}
In this section, we will apply the switching control method to an active magnetic bearing (AMB) system. Owing to the linear dependence of the plant dynamics on the rotor speed, the AMB plant can be simplified to a set of linear time-varying differential equations as [18]

$$
\dot{\theta} = -\frac{\rho J_s}{J_r} \psi + \frac{1}{m} (-4c_1 \theta + 2c_1 \phi \psi + f_{d\theta})
$$

$$
\dot{\psi} = \frac{\rho J_s}{J_r} \theta + \frac{1}{m} (-4c_2 \psi + 2c_1 \phi \psi + f_{d\phi})
$$

$$
\dot{\phi} = \frac{1}{N} (e_{\theta} + 2d_2 \theta - d_1 \phi)
$$

$$
\dot{\psi} = \frac{1}{N} (e_{\phi} + 2d_2 \psi - d_1 \phi)
$$

(34)

where $\rho$ denotes the rotor speed. $\theta, \psi$ are the Euler angles denoting the orientation of rotor centerline. $J_s, J_r$ are the moment of inertia of the rotor in axial and radial directions, respectively. $\phi, \psi$ are the differential magnetic flux from electromagnetic pairs, $e_{\theta}, e_{\phi}$ are the corresponding differences of electric voltage. $f_{d\theta}, f_{d\phi}$ are disturbance forces caused by gravity, modeling errors, imbalances, etc. The values of $c_1, c_2, d_1, d_2$ and $m$ depend on the AMB’s geometry and parameters, which can be found in [20].

Let $x^T = [\theta \quad \dot{\theta} \quad \psi \quad \dot{\psi} \quad \phi \quad \dot{\phi} \quad w^T = [f_{d\theta} \quad f_{d\phi} \quad ], \quad u^T = [e_{\theta} \quad e_{\phi} \quad ]$. In automatic balancing design, $f_{d\theta}, f_{d\phi}$ are typically modeled as sensor noise on the measured rotor displacement. Under this assumption, the linearized equations (34) can then be written as the following LPV system

$$
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} =
\begin{bmatrix}
A (\rho) & B_1 (\rho) & B_2 (\rho) \\
C_1 (\rho) & D_{11} (\rho) & D_{12} (\rho) \\
C_2 (\rho) & D_{21} (\rho) & D_{22} (\rho)
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
u
\end{bmatrix},
$$

where the state-space data are

$$
A (\rho) =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-\frac{4c_2}{m} & 0 & 0 & -\frac{1}{J_r} & 0 & \frac{2c_2}{m} \\
-\frac{4c_1}{m} & 0 & 0 & 0 & -\frac{1}{J_r} & \frac{2c_1}{m} \\
\frac{2c_1}{N} & 0 & 0 & 0 & 0 & -\frac{1}{N} \\
\frac{2c_2}{N} & 0 & 0 & 0 & 0 & -\frac{1}{N}
\end{bmatrix},
$$

$$
B_1 (\rho) = 0_{6 \times 2}, B_2 (\rho) = \frac{1}{N} \begin{bmatrix}
0_{4 \times 2} \\
I_2
\end{bmatrix},
$$

$$
C_1 (\rho) =
\begin{bmatrix}
I_2 & 0_{2 \times 4}
\end{bmatrix},
$$

$$
D_{11} (\rho) = 0_{4 \times 2}, D_{12} (\rho) = \begin{bmatrix}
0_{2 \times 2} \\
I_2
\end{bmatrix},
$$

$$
C_2 (\rho) =
\begin{bmatrix}
I_2 & 0_{2 \times 4}
\end{bmatrix}, D_{21} (\rho) = I_2, D_{22} (\rho) = 0_{2 \times 2}.
$$

The rotor speed $\rho$ is assumed to be available in real-time for control. The rotor speed is assumed to vary between $315$ and $1100\text{rad/s}$.

We observe that the matrices $B_1 (\rho), B_2 (\rho), C_1 (\rho), C_2 (\rho), D_{11} (\rho), D_{12} (\rho), D_{21} (\rho), D_{22} (\rho)$ are parameter independent. With the parameter $\rho$ vary in a polytope of two vertices $\omega_1 = 315$ and $\omega_2 = 1100$, the $A (\rho)$ ranges in the polytope $C_0 [A_n, n = 1, 2]$. The vertices $A_n$ are values of $A (\rho)$ at the two vertices of parameter area: $\omega_1, \omega_2$. Due to large variations of rotor speed, it could be conservative to use a single LPV controller over the entire parameter region. We
divide the parameter polytope into two areas [315, 720] and [720, 1100]. For each parameter area, the \( A(\rho) \) ranges in a smaller polytope \( Co\{A_{mn}, m = 1, 2; n = 1, 2\} \), where \( m \) is the \( m \)th divided area. We design two Lyapunov functions for the two parameter polytopes. The theorem condition in the previous section will be used for control design. To avoid solving the non-convex problem, we enforce the constraint \( R_i = R_j \). After solving the LMIs in theorem, we can get the weighted disturbance attenuation level \( \gamma \) in Table 1.

**Remark 2** The \( \gamma \) in Table 1 is the weighted disturbance attenuation level in the whole parameter region which is not divided. The \( \gamma_1 \) and \( \gamma_2 \) is the weighted disturbance attenuation level in the subregions [315, 720] and [720, 1100]. The \( \gamma_s \) is the maximum of \( \gamma_1 \) and \( \gamma_2 \). The values of \( \gamma \), \( \gamma_1 \) and \( \gamma_2 \) are all optimized by the method in [21].

We then conduct the simulation for AMB using two subregions switching LPV polytopic control. A time-varying rotor speed profile is chosen as in Fig.3. Note that the rotor speed trajectory is deliberately chosen to cross the intersection of two parameter subregions [315, 720] and [720, 1100] back and forth to illustrate the effect of LPV control switching. Disturbances \( f_{3\theta} \) and \( f_{4\psi} \) are chosen as impulsive inputs 0.002 and -0.0025. As shown in Fig.4, the switching occurs at 1.46s and 3.8s, respectively. The state response and control input of simulation are presented in Fig.1 and Fig.2 respectively.

**5 Conclusion**

Design of a switched LPV controller with multiple Lyapunov functions has been proposed in our research. For a
given LPV polytopic system, its performance is mainly deter-
determined by the choice of Lyapunov function. When pa-
parameters span a large range, designing of a single Lyapunov
function over the entire parameter set is conservative. We
divide the parameter polytopic region into subregions and
propose a sufficient LMI condition using multiple Lyapunov
functions method. A family of LPV polytopic controllers
are designed, and each is suitable for a specific parameter
region. The control strategy is applied to a magnetic active
bearing control system and promising simulation results are
obtained.

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