Passive Control for Networked Switched Systems with Network-Induced Delays and Packet Dropout

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Abstract—It has become a common practice to employ networks in control systems for connecting controllers and sensors/actuators on controlled plants and processes. A networked switched control system, as a special case of networked control systems, is studied. Such a system is represented with network-induced delays and packet dropout as a switched time-varying delay system. Sufficient conditions for exponential stability with strict passivity are derived for a class of switching signals with the average dwell time. Hybrid passive controller design depending on the network-induced delays and packet dropout is given in terms of linear matrix inequalities.

I. INTRODUCTION

NETWORKED switched control systems (NSCS), as a special class of networked control systems, whose switched plant with all sensors/actuators and hybrid controllers connected by communication channels, have drawn attention in the control and computer community recently [1-4]. This is mainly due to the switched and hybrid nature of many physical processes and the growing use of the communication network in control of the physical plants. Similar to networked control systems [5-8], this type of system allows for reduced wiring as well as for lower installation cost. It also permits greater agility in diagnosis and maintenance and flexibility for industrial field applications.

As we all know, a switched system consists of a family of continuous time subsystems and a rule that orchestrates the switching between them. The research is focused on finding stability/stabilizability conditions under an arbitrary switching rule or finding the constraints that should be imposed to the switching signal to guarantee the stability of the system [10-17]. The common Lyapunov function technique [9, 10], the multiple Lyapunov function technique [11-14] and the average dwell time technique [15-17] are used as effective tools in analyzing and designing switched systems. All the above results are obtained under the signal perfect transmission. However, for the network-based switched systems, the limitation of network bandwidth and the differences of various transmission protocols affect the transmission timing and accuracy. In particular, network-induced delays [5] and packet dropout [5, 8] often occur. Therefore, the existing effective methods for switched systems must be reevaluated before they become applicable to the network-based switched systems. How to design switching controllers and switching rules when switched system infrastructure is networked implementation has also posed a theoretical control problem.

Although there exist some results on the stability of networked switched systems, the control synthesis issue has not been fully investigated. The NSCS with network-induced delays in the system state under arbitrary switching signals have been proposed in the control literature. Stabilization of linear switched systems with network-induced delays which is less than one sample period has been studied in [2]. The NSCS with both the switching signal and control input affected by network-induced delays is discussed; linear matrix inequalities (LMI) control design based on an equivalent polytopic system with additive norm bounded uncertainty under arbitrary switching signals is also given. See, for instance, [1, 3], and the references therein. In [4], a class of hybrid multi-rate control models with a constant time delay and switching controllers is formulated and robust passivity analysis for the discrete system under an arbitrary switching signal is investigated. But the design of switching controllers is not mentioned. Moreover, one of the basic problems of switched systems is to design the most appropriate switching rule to stabilize the system even though all the subsystems are stable. Since the insertion of communication channel into the switched systems makes the analysis and design much more complicated, the stability condition for switched system with network-induced delays under arbitrary switching signals is obviously limited and conservative. How to design switching rules that take the network-induced delays and packet dropout into account is of great importance. This motivates our present work.

On the other hand, the passivity theory gives a framework for the design and analysis of control systems using an input-output description based on energy-related considerations [18, 22]. The passivity theory intimately is related to the circuit analysis method and plays an important role in both electrical network and control systems. This
mainly due to the fact that passivity and stability are closely related supplying a new method to solve the stabilization problem [19, 20]. Moreover, passivity-based control, as an energy-based analysis approach, has important robustness properties [21]. Therefore, in this paper, passivity-based control for networked switched systems with network-induced delays and packet dropout is studied.

The paper is organized as follows: a switched system with network-induced delays and packet dropout is formulated in Section II. Lyapunov-Krasovskii technique is used to analyze the NSCS. A network-dependent criterion on passivity of NSCS is given in Section III. Section IV gives the hybrid controller design depending on the network condition and a class of switching signals with the average dwell time in terms of linear matrix inequalities (LMIs). An example is given to illustrate our result in Section V. Finally, Section VI contains the conclusions.

II. PROBLEM FORMULATION

The system to be studied in this paper can be depicted in Fig. 1. A switched plant, via actuators and sensors, interacts with hybrid controllers through network channels. Thus, the closed-loop system can be modeled as a continuous-time linear switched system

\[ \ddot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)}(t) + \Gamma_{\sigma(t)}\omega(t) \]

(1)

\[ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t) \]

with hybrid state feedback controllers

\[ u_{\sigma(t)}(t) = K_{\sigma(t)}\dot{x}(t_k), \quad t \in [t_k, t_{k+1}) \]

(2)

where \( x(t) \in \mathbb{R}^n \) denote the switched plant state, \( z(t) \in \mathbb{R}^p \) is the output, \( \omega(t) \in \mathbb{R}^m \) is the exogenous disturbance input which belongs to \( L_2[0, \infty) \), \( u_{\sigma(t)}(t) \) is the hybrid control input, \( \sigma(t) : \mathbb{R}_+ = [0, \infty) \rightarrow Y = \{1, 2, ..., m\} \) is the switching signal which is a piecewise constant function depending on time \( t \) and/or state \( x(t) \). The switching signal \( \sigma(t) \) can be characterized by the following switching sequence

\[ \Sigma = \{x_0; (t_0, t_1), \ldots, (t_k, t_{k+1}), \ldots, i_k \in Y, k \in N\} \]

in which \( x_0 \) is the initial state, \( t_0 \) is the initial time and \( N \) is the set of nonnegative integers. The \( i_k \) th subsystem is activated when \( t \in [t_k, t_{k+1}) \). \( \sigma(t) = i_k \). \( u_i(t) \in \mathbb{R}^n \) is the control input of the \( i \) th subsystem, \( A_i, B_i, C_i, D_i, \Gamma_i \), \( \forall i \in Y \), are constant matrices; time \( t_k \) is the sampling instant; \( u_i(t) \), \( \forall i \in Y \), are piecewise continuous functions and only change the value at \( t_k \); \( K_i \), \( \forall i \in Y \) are controller gains; \( h \) is the sampling period, which is a positive scalar; and \( \dot{x}(t_k) \) denotes the state signal received via network transmission at the sampling instant \( t_k \).

As depicted in Fig. 1, if network congestion or node failure occurs, depending on the network protocol employed, network-induced delays and packet dropout happen inevitably. For the purpose of analysis, the whole delay at each sampling period is denoted by \( \tau = \tau_{sc,k} + \tau_{ca,k} \), where \( \tau_{sc,k} \) is the sensor-to-controller delay, and \( \tau_{ca,k} \) is the controller-to-actuator delay [5]. The received signals by the hybrid controllers can be conveniently described as follows [6]:

\[ \dot{x}(t_k) = x(t_k), \quad \text{if no packet dropout at time } t_k; \]

\[ \dot{x}(t_k) = x(t_k - h), \quad \text{if one packet dropout at time } t_k; \]

\[ \dot{x}(t_k) = x(t_k - n(k)h), \quad \text{if } n(k) \text{ packets dropout at time } t_k. \]

Therefore, network-induced delays and packet dropout can be lumped into a generalized description

\[ \dot{x}(t_k) = x(t_k - n(k)h - \tau_k) \]

(3)

Replacing \( t - (t_k - n(k)h - \tau_k) \) by a new variable \( r(t) \), we have

\[ \dot{x}(t) = x(t - r(t)) \]

(4)

where \( r(t) \) is time-varying delay satisfying

\[ 0 < r(t) = t - t_k + n(k)h + \tau_k \leq \tau_{\text{max}}, \quad t \in [t_k, t_{k+1}) \]

(5)

\[ \tau(t) = 1 \]

(6)

where \( \tau_{\text{max}} > 0 \) is the upper bound of delay. The condition (6) is obvious from the condition (5), which is characteristic of many communication networks. See the reference [7].

The closed-loop NSCS can thus be described by

\[ \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}x(t - r(t)) + \Gamma_{\sigma(t)}\omega(t), \quad t \in [t_k, t_{k+1}) \]

\[ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t) \]

(7)

where the switching signal \( \sigma(t) \) and the hybrid controller gains \( K_i \), \( \forall i \in Y \), are to be designed, and
\(x(\theta) = \phi(\theta), \ \theta \in [-\tau_{\text{max}}, 0]\), where \(\phi(\theta) \in C_n\) is a differentiable vector-valued initial function on \([-\tau_{\text{max}}, 0]\).

We assume the state of system (7) is continuous at the switching instant. Also \(\sigma(t)\) has finite number of switchings on any finite interval of time. These assumptions are standard in the switched system literatures [9-10, 14-17].

The objective of this paper is to design hybrid state feedback controllers such that system (7) is exponentially stable with strict passivity for all admissible network-induced delays and packet dropout. That is

(i) The networked switched systems are exponentially stable for any network-induced delays and packet dropout.

(ii) Under the zero initial condition, there exists a scalar \(\beta > 0\) such that

\[
2\int_0^T \omega^T(s)\omega(s)ds \geq \beta \int_0^T \omega^T(s)\omega(s)ds
\]

hold for all \(T > 0\) and any \(\omega(t) \in L^2[0, T]\).

III. NETWORK-DEPENDENT PASSIVITY

In the sequel, the network-dependent passivity of NSCS (7) is to be discussed.

**Definition 1** [17]. The equilibrium \(x^* = 0\) of system (7) is said to be exponentially stable under the switching signal \(\sigma(t)\), if the solution of system (7) satisfies

\[
\|x(t_0, \phi)(t)\| \leq \lambda \|\phi\|_0 e^{-\gamma(t-t_0)}, \ \forall t \geq t_0
\]

for some constants \(\lambda \geq 1\) and \(\gamma > 0\), where \(\|\cdot\|\) denotes the Euclidean norm, and \(\|\cdot\|_0 = \sup_{t \geq 0} \|x(t)\|\).

We denote the number of discontinuities of switching signal \(\sigma(t)\) on an interval \([t, T]\) by \(N_\sigma(t, T)\). \(\sigma(t)\) is said to have an average dwell time \(\tau_a\), if there exist two numbers \(N_0 \geq 0\) and \(\tau_a > 0\) such that

\[
N_\sigma(t, T) \leq N_0 + \frac{T - t}{\tau_a}, \forall T \geq t \geq 0.
\]

The following Lemma will be used in the proof of our main results.

**Lemma 1** [23] (Moon’s inequality): For any \(Z \in \mathbb{R}^{2n \times 2n}\), \(a \in \mathbb{R}^n, b \in \mathbb{R}^n, c \in \mathbb{R}^n\), \(W \in \mathbb{R}^{2n \times 2n}, \ Y \in \mathbb{R}^{n \times 2n}\), the following inequality holds:

\[
-2b^TNa \leq \begin{bmatrix} a^T \\ b \end{bmatrix} \begin{bmatrix} W & Y - N^T \\ Y^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix},
\]

where

\[
\begin{pmatrix} W & Y \\ * & Z \end{pmatrix} \geq 0.
\]

**Theorem 1.** For the given constants \(\alpha > 0, \tau_{\text{max}} > 0\), \(\beta > 0\) and the controller gain matrices \(K_i\), suppose that (5)-(6) hold. If there exist matrices \(P_{i_1} > 0, P_{i_2}, P_{i_3}, Q_i > 0, Z_i, R_i > 0\) such that the inequalities

\[
\begin{pmatrix} \Pi_i & P_{i_1}^T \\ P_{i_2} & \Gamma_i \end{pmatrix} < 0
\]

\[
\begin{pmatrix} \Pi_i & P_{i_1}^T \\ P_{i_2} & \Gamma_i \end{pmatrix} < 0
\]

\[
\begin{pmatrix} e^{-\alpha t_{\text{max}}} R_i & 0 \\ 0 & K_i^T B_i^T \end{pmatrix} P_i \geq 0
\]

hold for \(\forall i \in \mathbb{Y}\), then NSCS (7) is exponentially stable with strict passivity for any switching signal with the average dwell time satisfying

\[
\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}.
\]

where \(\mu \geq 1\) satisfies

\[
P_{i_1} \leq \mu P_{j_1}, Q_i \leq \mu Q_j, R_i \leq \mu R_j, \forall i, j \in \mathbb{Y},
\]

\[
\Pi_i = P_{i_1}^T \tilde{A}_i + \tilde{A}_i^T P_{i_2} + \tau_{\text{max}} Z_i + \begin{bmatrix} Q_i & 0 \\ 0 & \tau_{\text{max}} R_j \end{bmatrix} + \alpha E P_i,
\]

\[
\tilde{A}_i = \begin{bmatrix} 0 & I \\ A_i + B_i K_i & -I \end{bmatrix}, P_i = \begin{bmatrix} P_{i_1} & 0 \\ P_{i_2} & P_{i_3} \end{bmatrix},
\]

\[
Z_i = \begin{bmatrix} Z_{i_1} & Z_{i_2} \\ * & Z_{i_3} \end{bmatrix}.
\]

**Proof.** System (7) can be converted into an equivalent descriptor form similar to the one in the reference [24]:

\[
\dot{x}(t) = y(t)
\]

\[
\begin{pmatrix} -y(t) + A_{\sigma(t)} x(t) \\ + B_{\sigma(t)} u_{\sigma(t)}(t) + \Gamma_{\sigma(t)} \omega(t) \end{pmatrix}, \ \text{if } t \in (0, T_{\text{max}})
\]

\[
0 = \begin{pmatrix} -y(t) + (A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma(t)}) x(t) \\ -B_{\sigma(t)} K_{\sigma(t)} \int_{t-T_{\text{max}}}^{t} y(s)ds + \Gamma_{\sigma(t)} \omega(t) \end{pmatrix}, \ \text{if } t \geq T_{\text{max}}
\]

\[
x(t) = C_{\sigma(t)} x(t) + D_{\sigma(t)} \omega(t)
\]

with \(x(\theta) = \phi(\theta), \ \theta \in [-\tau_{\text{max}}, 0]\) being the initial condition.

We choose the piecewise Lyapunov-Krasovskii functional candidate as follows

\[
V(x(t)) = V_{\sigma(t)}(x(t)) = V_{\sigma_1(t)}(x(t)) + V_{\sigma_2(t)}(x(t)) + V_{\sigma_3(t)}(x(t))
\]

where

\[
V_{\sigma_1(t)}(t) = s^T(t) E P_{\sigma(t)} s(t),
\]

\[
V_{\sigma_2(t)}(t) = \int_{-T_{\text{max}}}^{0} \int_{t}^{t_0} y^T(s) e^{\sigma(t-s)} R_{\sigma(t)} y(s)dsdt,
\]

\[
V_{\sigma_3(t)}(t) = \int_{t-T_{\text{max}}}^{t} x^T(s) e^{\sigma(t-s)} Q_{\sigma(t)} x(s)ds
\]

and \(Q_i, R_i, \forall i \in \mathbb{Y}\), are positive definite matrices, and also
Differentiating $V_i(x_i)$ along the trajectory of system (13) results in
\[ \dot{V}(x_i) + \alpha V(x_i) - 2\omega^T(t)z(t) + \beta \omega^T(t)\omega(t) = s^T(t)(P_i^T \bar{A}_i + \bar{A}_i^T P_i)s(t) + 2s^T(t)P_i^T 0 \Gamma_i \omega(t) \]
\[ + \eta_i + \tau_{\text{max}} y^T(t) R_i y(t) + s^T(t)\alpha EP_i x(t) \]
\[ - \int_{t-\tau_{\text{max}}}^t y^T(s)e^{-\alpha(s-t)} R_i y(s)ds + x^T(t)Q_i x(t) \]
\[ - 2\omega^T(t)z(t) + \beta \omega^T(t)\omega(t) \]
where
\[ \bar{A}_i = \begin{pmatrix} 0 & I \\ A_i + B_i K_i & -I \end{pmatrix} \]
\[ \eta_i = -2s^T(t)P_i^T \begin{pmatrix} 0 \\ B_i K_i \end{pmatrix} \int_{t-\tau(t)}^t y(s)ds. \]
From Lemma 1, taking
\[ N_i = P_i^T \begin{pmatrix} 0 \\ B_i K_i \end{pmatrix}, b = s(t), a = y(s), \]
we obtain
\[ \eta_i \leq -2s^T(t)P_i^T \begin{pmatrix} 0 \\ B_i K_i \end{pmatrix} \int_{t-\tau(t)}^t y(s)ds \]
\[ \leq \int_{t-\tau(t)}^t y^T(s)(e^{-\alpha\tau_{\text{max}}} R_i Y_i - N_i^T Z_i) y(s)ds \]
where $R_i, Y_i, Z_i$ are constant matrices with appropriate dimensions such that $(e^{-\alpha\tau_{\text{max}}} R_i Y_i - N_i^T Z_i) \geq 0$ hold. Choosing
\[ Y_i = [0 \ K_i B_i] P_i \]
yields
\[ \eta_i \leq \int_{t-\tau(t)}^t y^T(s)e^{-\alpha\tau_{\text{max}}} R_i y(s)ds + \tau_{\text{max}} s^T(t)Z_i s(t) \]
Let
\[ \xi^T(t) = \begin{pmatrix} x^T(t) & y^T(t) & \omega^T(t) \end{pmatrix} \]
Then combining (15) and (18) leads to
\[ \dot{V}(x_i) + \alpha V(x_i) - 2\omega^T(t)z(t) + \beta \omega^T(t)\omega(t) \leq \xi^T(t)P_i \xi(t), \]
where
\[ \Phi_i = \begin{pmatrix} \Pi_i & P_i^T \begin{pmatrix} 0 \\ \Gamma_i \end{pmatrix} - C_i^T \\ * & -(D_i + D_i^T)^T + \beta I \end{pmatrix} \]
\[ \Pi = P \bar{A} + \bar{A} P + \tau Z + \begin{pmatrix} \tau R & 0 \\ 0 & \alpha EP \end{pmatrix}. \]
If $\Phi_i < 0$, then
\[ \dot{V}(x_i) + \alpha V(x_i) - 2\omega^T(t)z(t) + \beta \omega^T(t)\omega(t) < 0. \]

Since $V(x) \geq 0, \alpha > 0$, it holds that
\[ \dot{V}(x_i) + \alpha V(x_i) - 2\omega^T(t)z(t) + \beta \omega^T(t)\omega(t) \leq 0. \]

Under the zero initial condition, we have
\[ 2\int_0^T \omega^T(s)\omega(s)ds \geq \beta \int_0^T \omega^T(s)\omega(s)ds, \forall \omega \in \mathbb{L}_2[0, \infty) \]
According to (10)-(11), we know each subsystem of system (7) with $\omega(t) = 0$ is exponentially stable for $\forall i \in \mathbb{Y}$. Thus for $t \in [t_k, t_{k+1})$, we have
\[ V(x_i) = V_{\sigma(t)}(x_{tk}) < e^{-\alpha(t-t_k)}V_{\sigma(t_k)}(x_{tk}) \]
When $\omega(t) = 0$, from (10)-(11), at switching instant $t_k$, we obtain
\[ V_{\sigma(t_k)}(x_{tk}) \leq \mu V_{\sigma(t_{tk})}(x_{tk}) \]
Therefore, given the switching number
\[ N_0 \leq N_0 + \frac{t-t_0}{\tau_a}, \]
it follows from (22)-(23) that
\[ V(x_i) \leq e^{-\alpha(t-t_k)}\mu V_{\sigma(t_{tk})}(x_{tk}) \]
\[ \leq e^{-\alpha(t-t_k)} \mu e^{-\alpha(t_{tk}-t_{k-1})} V_{\sigma(t_{k-1})}(x_{k-1}) \]
\[ \leq \cdots \leq e^{-\alpha(t-t_0)} \mu N_0 e^{-\alpha(t-t_0)} V_{\sigma(t_0)}(x_{t_0}) \]
\[ \leq \mu N_0 e^{-\alpha(t-t_0)} V_{\sigma(t_0)}(x_{t_0}) \]
Since $\|x(t)\|^2 \leq V(x_i)$ and $V(x_{t_0}) \leq b \|x_{t_0}\|^2$, we infer
\[ \|x(t)\|^2 \leq \frac{1}{a} V(x_i) \leq \frac{b}{a} \mu N_0 e^{-\alpha(t-t_0)} \|x_{t_0}\|^2 \]
where
\[ a = \min_{\forall i \in \mathcal{Y}} \lambda_{\min}(P_i), \]

\[ b = \max_{\forall i \in \mathcal{Y}} \lambda_{\max}(P_i) + \max_{\forall i \in \mathcal{Y}} \lambda_{\max}(Q_i) + \frac{r_{\max}^2}{2} \max_{\forall i \in \mathcal{Y}} \lambda_{\max}(R_i), \]

which completes the proof.

**Remark 2.** Theorem 1 gives a sufficient condition for the exponential stability with strict passivity of NSCS (7) by inequalities (10)-(11). The inequalities (10)-(11) hold depending on an important parameter \( r_{\max} \), which describes the upper bound of network-induced delays and packet dropout (see inequality (5) for details). Since the parameter \( r_{\max} \) and (5) depend on the network, we say the inequalities (10)-(11) with the parameter \( r_{\max} \) are network-dependent conditions.

IV. DESIGN OF HYBRID STATE FEEDBACK PASSIVE CONTROLLERS

Now, we are concerned with the design of hybrid state feedback controllers which render the closed-loop system exponentially stable with strict passivity for all admissible network-induced delays and packet dropout.

**Theorem 2.** For the given scalars \( \alpha > 0 \), \( \tau_{\max} > 0 \) and \( \beta > 0 \), suppose that there exist matrices \( M_{i1}, M_{i2}, M_{i3}, Z_{i1}, Z_{i2}, Z_{i3}, R_i > 0, \hat{Q}_i > 0, \hat{X}_i \) such that the following linear matrix inequalities

\[
\begin{bmatrix}
\psi_1 & -M_{i1}^T C_i^T & \tau_{\max} M_{i1}^T & M_{i1}^T \\
* & \psi_3 & \Gamma_i & \tau_{\max} M_{i3}^T & 0 \\
* & * & -(D_i + D_i^T) + \beta I & 0 & 0 \\
* & * & * & -\tau_{\max} \hat{Q}_i & 0 \\
* & * & * & * & -\hat{Q}_i
\end{bmatrix} < 0 \quad (26)
\]

\[
\begin{bmatrix}
e^{-\alpha \tau_{\max}} (M_{i1} + M_{i1}^T - \hat{R}_i) & 0 & \hat{X}_i^T B_i^T \\
* & \hat{Z}_{i1} & \hat{Z}_{i2} \\
* & * & \hat{Z}_{i3}
\end{bmatrix} \geq 0 \quad (27)
\]

hold for \( \forall i \in \mathcal{Y} \). Then hybrid state feedback controllers (2) for any switching signal with the average dwell time satisfying (12) exponentially stabilize NSCS (7) with strict passivity for any network-induced delays and packet dropout satisfying (5) and (6). Moreover, the hybrid state feedback passive controller gains are

\[ K_i = \hat{X}_i M_{i1}^{-1}, \forall i \in \mathcal{Y}, \]

where \( \mu \) is a constant satisfying

\[ M_{i1}^{-1} \leq \mu M_{j1}^{-1}, \hat{Q}_i^{-1} \leq \mu \hat{Q}_j^{-1}, \hat{R}_i^{-1} \leq \mu \hat{R}_j^{-1}, \forall i, j \in \mathcal{Y}, \]

\[ \psi_1 = M_{i1} + M_{i2}^T + \tau_{\max} \hat{Z}_{i1} + \alpha M_{i1}^T, \]

\[ \psi_2 = M_{i3} + M_{i2}^T + M_{i3} A_i^T + \hat{X}_i^T B_i^T + \tau_{\max} \hat{Z}_{i2}, \]

\[ \psi_3 = -M_{i3}^T - M_{i3} + \tau_{\max} \hat{Z}_{i3} \]

**Proof.** Define

\[ M_i = P_i^{-1} = \begin{pmatrix} M_{i1} & 0 \\ M_{i2} & M_{i3} \end{pmatrix}, \quad \hat{Z}_i = M_i^T Z_i M_i = \begin{pmatrix} \hat{Z}_{i1} & \hat{Z}_{i2} \\ \hat{Z}_{i2} & \hat{Z}_{i3} \end{pmatrix}, \]

\[ \hat{R}_i = R_i^{-1}, \hat{Q}_i = Q_i^{-1}, \hat{X}_i = K_i M_{i1} \]

Upon carrying out the multiplications by \( M_i^T \) and its transpose on the left and right sides of (10) respectively, the application of Schur complement formula yields (26).

By multiplying with \( \text{diag}\{M_{i1}, M_i^T\} \) and its transpose on the left and right sides of (11), we obtain

\[
\begin{pmatrix}
M_{i1} e^{-\alpha \tau_{\max}} \hat{R}_i^{-1} M_{i1}^T & 0 & \hat{Y}_i^T B_i^T \\
* & \hat{Z}_{i1} & \hat{Z}_{i2} \\
* & * & \hat{Z}_{i3}
\end{pmatrix} \geq 0 \quad (28)
\]

Since \( \hat{R}_i = R_i^{-1} > 0 \), we have

\[
( M_{i1} - \hat{R}_i ) e^{-\alpha \tau_{\max}} \hat{R}_i^{-1} ( M_{i1} - \hat{R}_i )^T > 0,
\]

which implies

\[ M_{i1} e^{-\alpha \tau_{\max}} \hat{R}_i^{-1} M_{i1} \geq e^{-\alpha \tau_{\max}} ( M_{i1} + M_{i1}^T - \hat{R}_i ) \quad (29) \]

Hence (27) is guaranteed by inequalities (29), which completes the proof.

V. AN EXAMPLE

In this section, an example is given to illustrate our main results.

Consider the following switched system with two subsystems whose feedback paths are implemented by network.

Subsystem 1:

\[
\begin{align*}
\dot{x} &= (-0.3 \ 0.1) x + (0 \ 1) u + (-1 \ 0.2) \omega(t) \\
z &= (-1 \ 0.5) x + \omega(t)
\end{align*}
\]

Subsystem 2:

\[
\begin{align*}
\dot{x} &= (0.1 \ 0) x + (1 \ -1) u + (0.3 \ -2) \omega(t) \\
z &= (-1 \ 0.5) x + \omega(t)
\end{align*}
\]

where \( \omega(t) \in L_2 \).

Suppose that the network bandwidth is limited and the network-induced delays and packet dropout, which are bounded, and we describe them as equation (5). Given \( r_{\max} = 0.3, \alpha = 0.3, \beta = 0.2 \), solving LMIs (26) and (27) gives the hybrid state feedback controllers with gains
\[ K_1 = \bar{X}_1 M_{11}^{-1} = (1.2393 \ -2.0461), \]
\[ K_2 = \bar{X}_2 M_{21}^{-1} = (1.7305 \ -0.7131). \]

Therefore, we obtain the switching signal for guaranteeing the exponential stability with strict passivity of the whole switched systems satisfying \( \tau_a > \tau_a^* = \frac{\ln \mu}{\alpha} = 3.1658s \). With the initial state \( x(0) = (-2, 1)^T \), the state trajectories of system (7) and the switching signal are shown in Fig.2 and Fig.3, respectively.

![Fig.2 The state response of networked switched system](image)

![Fig.3 The switching signal with average dwell time 3.1658s](image)

VI. CONCLUSIONS

We have studied passive control for a class of networked switched systems. The linear switched systems and hybrid state feedback controllers are connected via network transmission channels which are subjected to network-induced delays and packet dropout. The proposed solution is found by using descriptor system representation. We have derived network-dependent conditions on exponential stabilization with strict passivity of NSCS under a class of switching signals with the average dwell time. The hybrid passive controllers stabilize the switched systems with a given maximal tolerance of network-induced delays and packet dropout represented by the upper bound of time-varying delay.

REFERENCES


