A State Feedback $H_\infty$ Control Design for Switched Fuzzy Systems

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Abstract—A model of the switched fuzzy systems whose subsystems are fuzzy systems is presented first. Then the problem of state feedback robust $H_\infty$ control of this class of systems is solved using switching technique and single Lyapunov function. A switching strategy of the switched fuzzy system with continuous controlled input is developed. The main condition is given in form of convex combination linear matrix inequalities, which are solvable using LMI Toolbox. The elaborated illustrative but real-world example and the respective simulation experiments demonstrate the effectiveness of the proposed method.

Index Terms—Fuzzy systems, intelligent control, nonlinear systems, switched systems.

I. INTRODUCTION

During the last decade switched systems [1 - 4] have attracted considerable research attention, whereas fuzzy systems have been investigated for quite some time and fuzzy system based intelligent controls have gained momentum. The importance of fuzzy systems has been long established but switched systems quickly appeared to be a rather important kind of hybrid systems [5 - 7]. Therefore, recently research endeavors to exploit the synergy of fuzzy system and switched systems theories. In turn, a novel research direction in control of dynamic systems is emerging towards developing new control designs employing that synergy.

It should be noted, a switched system consists of a number of sub-systems, continuous-time and/or discrete-time dynamic systems, and a switching law that orchestrates the switching between the sub-systems. The applications in computer disc drives [5], some robot control systems [6], the cart-pendulum systems [7], and other engineering systems indicated that switched systems have extensive practice background. Therefore, it has both theoretical significance and practical value to study switched systems.

On the other hand, fuzzy logic control has emerged as one of the most active and fruitful areas. Recently, some useful stability analysis techniques have come forth. LMI-based designs for T-S fuzzy systems have stimulated a considerable orientation toward studying fuzzy control techniques [8]. Moreover, LMI techniques are employed to solve an $H_\infty$ control problem of a nonlinear control system via robust $H_\infty$ fuzzy control [9]. The quadratic stabilization of uncertain fuzzy systems by state feedback has been also considered in [10] using $H_\infty$, Riccati inequalities or linear matrix inequalities (LMIs). The $H_\infty$ control problem for uncertain discrete-time fuzzy systems by state feedback has been considered in [11]. In particular, the mixed $H_2/H_\infty$ fuzzy feedback control problems by using LMIs have been considered in work [12].

A switched system is said to be a switched fuzzy system if all of its sub-systems are fuzzy systems. This class of systems can often more precisely describe continuous dynamics and discrete dynamics as well as their interactions in actual real-world systems. Compared with the results on stability of switched systems and those of fuzzy control systems, the results on switched fuzzy systems are very few. In [13], the combination of hybrid systems and fuzzy multiple model systems is described, and a fuzzy switched hybrid controller has been put forward. In [14, 15], a switching fuzzy model has been studied. Such a switching fuzzy system model has two levels of structure, which the first level is region rule level and the second level is local fuzzy rule level. This model is switching in local fuzzy rule level of the second level according to the premise variable in region rule level of the first level. Stability conditions are given [16 - 18] that gives some extension based on [14, 15].

A new model of class of switched fuzzy systems is proposed in this paper, which differs from existing ones known so far. A system of this class is a switched system whose subsystems are all fuzzy systems. The design method inherits some hybrid features and exploits the information of fuzzy systems. In contrast with the existing results, we study switched fuzzy system without levels of structure. The method provides a kind of different premise variable switching directly while [14 - 18] have considered the model with two levels of structure. We design both the continuous controllers for subsystems, based on state feedback $H_\infty$ robust control, and the switching law to bind them together. Furthermore, based on the single Lyapunov function technique, a sufficient condition for the switched fuzzy...
systems to be asymptotically stable with H\textsubscript{\infty} norm bound is derived.

In the paper, Section II presents the model development study. Section III presents the main result of this control design research. In Section IV the method is applied to design a room air-temperature regulating system. Conclusion and references follow thereafter.

II. SYSTEM MODEL

The overall representation of the class of continuous-time switched fuzzy system model with uncertainties (every subsystem is assumed to be an uncertain fuzzy system) in fuzzy system formalism can be given in terms of the following rule base:

\[ R_{\sigma(t)}^I : \text{if } \xi_1 \in M_{\sigma(t)}^I \ldots \text{and } \xi_p \in M_{\sigma(t)}^P, \text{ then} \]

\[ \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}w_{\sigma(t)}(t) + B_{z\sigma(t)}u_{\sigma(t)}(t), \]

\[ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}u_{\sigma(t)}(t), \quad l = 1,2,\ldots,N_{\sigma(t)}, \tag{1} \]

with \( R_{\sigma(t)}^I \) denoting the \( l \)-th fuzzy inference rule in the \( \sigma \)-th switched sub-system, whereas \( N_{\sigma(t)} \) is the number of inference rules in the \( \sigma \)-th switched sub-system. Note that fuzzy rules are selected in every switched sub-system. In there, variables denote: \( \xi_1, \xi_2, \ldots, \xi_p \) are the premise variables; \( \sigma(t) : R_+ \to M = \{1,2,\ldots,m\} \) is a piecewise constant function that is called switching signal; quantities \( M_{\sigma(t)}^I, \ldots, M_{\sigma(t)}^P \) denote fuzzy sets in the \( \sigma \)-th switched subsystem; \( u_{\sigma(t)}(t) \) is the actual control input of the \( \sigma \)-th switched subsystem; \( x(t) \) is the state variable vector; \( z(t) \) is the output to be regulated; and \( w_{\sigma(t)}(t) \) is disturbance input of the \( \sigma \)-th switched subsystem. Matrices \( A_{\sigma(t)}^I, B_{\sigma(t)}^I, B_{z\sigma(t)}^I \) and \( C_{\sigma(t)}, D_{\sigma(t)} \) are known constant matrices of appropriate dimensions for the \( \sigma \)-th switched sub-system.

A closer consideration of the rule base reveals the \( i \)-th switched subsystem is described by the following rule base of Takagi-Sugeno (T-S) category:

\[ R_{i}^I : \text{if } \xi_1 \in M_{i}^I \ldots \text{and } \xi_p \in M_{i}^P, \text{ then} \]

\[ \dot{x}(t) = A_{i}x(t) + B_{i}w_i(t) + B_{zj}u_i(t), \]

\[ z(t) = C_{i}x(t) + D_{i}u_i(t), \quad l = 1,2,\ldots,N_i, i = 1,2,\ldots,m. \tag{2} \]

Thus, in turn, the model of the \( i \)-th switched subsystem is described by means of the following set of equations:

\[ \dot{x}(t) = \sum_{l=1}^{N_i} \eta_i(\xi_i(t)) \left[ A_{i}x(t) + B_{i}w_i(t) + B_{zj}u_i(t) \right], \]

\[ z(t) = \sum_{l=1}^{N_i} \eta_i(\xi_i(t)) \left[ C_{i}x(t) + D_{i}u_i(t) \right], \quad i = 1,2,\ldots,m. \tag{3} \]

In this representation model (3), the quantities and the respective relationships that appear are as follows:

\[ 0 \leq \eta_i(\xi_i(t)) \leq 1, \quad \sum_{i=1}^{N} \eta_i(\xi_i(t)) = 1, \tag{4-a} \]

\[ w_i(t) = \prod_{\rho=1}^{p} M_{i\rho}^P(\xi_{\rho}(t)), \tag{4-b} \]

\[ \eta_i(\xi_i(t)) = \prod_{l=1}^{N_i} w_i(t), \tag{4-c} \]

where \( M_{i\rho}^P(\xi_{\rho}(t)) \) denotes the membership function, and \( \xi_{\rho}(t) \) belongs to the fuzzy set \( M_{i\rho}^P \).

Now, with the representation (3)-(4) at hand, the \( H_{\infty} \) control problem for the switched fuzzy system (1) can be formulated as follows:

Find a continuous state feedback controller \( u_i = u_i(x) \) for each subsystem and a switching law \( i = \sigma(t) \) such that:

(1) The closed-loop system is asymptotically stable when \( w_i = 0 \).

(2) The output \( z \) satisfies \( \|w_i\|_{2} < \gamma \|w_i\|_{2} \) under the zero initial condition, where constant \( \gamma > 0 \) is assumed given.

Remark 1: Switched systems partition the whole state space \( \mathbb{R}^n \) into \( m \) sub areas \( \Omega_1, \ldots, \Omega_m \), and every sub area is a switched sub-system. Switching among sub systems via the respective appropriate switching law is carried out to ensure stability of switched systems. Fuzzy systems partition the state space into many fuzzy sub-areas, and a local model is designed in every fuzzy sub-area. Global model of fuzzy system is composed of a series of local model which that are linked by fuzzy membership function.

When sub-systems of the switched system are T-S fuzzy systems, such systems in fact are switched fuzzy systems. A sketch map of the switched fuzzy systems in operating mode
is depicted in Figure 1. In there, $\Omega_l$ denotes the area of states state of the $i$-th switched subsystem, and $\Omega_{il}$ denotes the $l$-th fuzzy sub-area in $\Omega_l$. The switched fuzzy system partitions again the $\Omega_l$ into $l$ fuzzy sub-areas $\Omega_{i1}, \ldots, \Omega_{il}, \ldots, \Omega_{in}$. There is a local linear model in every fuzzy sub-area; namely, the local linear model in $\Omega_{il}$ is $$\dot{x}(t) = A_{il}x(t) + B_{il}u_i(t).$$
The model for every switched sub-area $\Omega_{i1}, \ldots, \Omega_{in}$ is composed of an appropriate linear model which is linked by fuzzy membership function.

In addition to the state feedback for the local linear models, also we design the switching law for the area fuzzy model to ensure stability of the switched fuzzy system. When local model in a given fuzzy-system area satisfies the switching law, then the switching to the $\Omega_{2l}$-th sub-system takes place to ensure stability of the switched fuzzy system as a whole.

### III. The Main Result

This section derives a condition for the $H_\infty$ control problem to be solvable as an LMI problem, and presents the designs of continuous controllers for subsystems and a switching law.

Here, the design method based on PDC fuzzy controller being used for every fuzzy sub-system. Namely, fuzzy controller and system (2) have the same fuzzy inference premise variables

$$R^l_{ij} : \xi_1 \mbox{ is } M^l_{ij} \mbox{ and } \xi_p \mbox{ is } M^l_{pj}, \mbox{ then } u_i(t) = K_{il}x(t), \quad l = 1, \ldots, N_j, \quad i = 1, \ldots, m, \quad \mbox{and the overall control is given as follows:}$$

$$u_i(t) = \sum_{j=1}^{N_j} \eta_{il}K_{ij}x(t). \quad (5)$$

Then the overall model representing the $i$-th fuzzy sub-system is described by:

$$\dot{x}(t) = \sum_{j=1}^{N_j} \eta_{il} \sum_{p=1}^{N_p} [A_{ij}x(t) + B_{ij}w_j + B_{2ij}K_{pj}x(t)]$$

$$z(t) = \sum_{j=1}^{N_j} \eta_{il} \sum_{p=1}^{N_p} (C_{ij} + D_{ij}K_{pj})x(t). \quad (6)$$

**Lemma.** Let $a_{ij}$ (1 ≤ $i \leq m, 1 \leq j \leq N_j$) be a group of constants satisfying

$$\sum_{i=1}^{m} a_{ij} < 0, \quad \forall 1 \leq j \leq N_j.$$  

Then, there exists at least one $i$ such that $a_{ij} < 0, 1 \leq j \leq N_j$.

**Proof:** Trivial hence omitted.

**Theorem 1:** Let a constant $\gamma > 0$ be given. Suppose there exist a positive definite matrix $P$ and constants $\lambda_{ij} > 0$ ($i = 1, 2, \ldots, m; j_i = 1, 2, \ldots, N_i$), such that

$$(A_{ij} + B_{2ij}K_{ij})^T P + P(A_{ij} + B_{2ij}K_{ij}) + \sum_{i=1}^{m} \lambda_{ij} \frac{1}{\gamma^2} PB_{ij} B_{ij}^T P + (C_{ij} + D_{ij}K_{ij})^T \bullet x < 0,$$

$$i = 1, 2, \ldots, m; j_i, \vartheta_j, p_j, q_j = 1, 2, \ldots, N_j. \quad (7)$$

Then the state feedback controllers (5) and the switching law (8), given below, solve the $H_\infty$ control problem formulated

$$\sigma(x) = \arg \min_{j_i, \vartheta_i, p_i, q_i} \frac{\lambda}{\gamma} \max \left\{ \frac{1}{x^T P x} \right\}$$

$$\begin{bmatrix}
(A_{ij} + B_{2ij}K_{ij})^T P + P(A_{ij} + B_{2ij}K_{ij}) + & \sum_{i=1}^{m} \lambda_{ij} \frac{1}{\gamma^2} PB_{ij} B_{ij}^T P + (C_{ij} + D_{ij}K_{ij})^T \bullet x < 0, \\
(C_{ij} + D_{ij}K_{ij})^T \bullet j_i, \vartheta_j, p_j, q_j = 1, 2, \ldots, N_j & 
\end{bmatrix} \quad (8)$$

while ensuring quadratic asymptotic stability.

**Proof:** From inequalities (7) we know that for any $x \neq 0$, it holds

$$(A_{ij} + B_{2ij}K_{ij})^T P + P(A_{ij} + B_{2ij}K_{ij}) + \sum_{i=1}^{m} \lambda_{ij} \frac{1}{\gamma^2} PB_{ij} B_{ij}^T P + (C_{ij} + D_{ij}K_{ij})^T \bullet x < 0,$$

$$i = 1, 2, \ldots, m; j_i, \vartheta_j, p_j, q_j = 1, 2, \ldots, N_j. \quad (9)$$

Note that (9) holds for any $j_i, \vartheta_j, p_j, q_j \in \{1, 2, \ldots, N_j\}$ and $\lambda_{ij} > 0$, the Lemma says that there exists at least one $i$ such that for any $j_i, \vartheta_j, p_j, q_j$ and $\lambda_{ij} > 0$, the Lemma says that there exists at least one $i$ such that for any $j_i, \vartheta_j, p_j, q_j$, it holds

$$\begin{bmatrix}
(A_{ij} + B_{2ij}K_{ij})^T P + P(A_{ij} + B_{2ij}K_{ij}) + & \sum_{i=1}^{m} \lambda_{ij} \frac{1}{\gamma^2} PB_{ij} B_{ij}^T P + (C_{ij} + D_{ij}K_{ij})^T \bullet x < 0, \\
(C_{ij} + D_{ij}K_{ij})^T \bullet j_i, \vartheta_j, p_j, q_j = 1, 2, \ldots, N_j & 
\end{bmatrix} \quad (10)$$

Thus, the switching law defined by (8) is well-defined.

Next we investigate the stability issue via employing the
quadratic Lyapunov function \( V(x(t)) = x^T(t)Px(t) \). The first derivative along systems state trajectories is found to be:

\[
\dot{V} = x^T(t)Px + x^T(t)P\dot{x}
\]

\[
= \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \left( A_{ir}x + B_{ir}w_i + B_{2ir}K_{ir}x \right)^T P x + \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T P \left( A_{ir}x + B_{ir}w_i + B_{2ir}K_{ir}x \right)
\]

\[
= \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T \left[ \left( A_{ir} + B_{2ir}K_{ir} \right)^T P + P \left( A_{ir} + B_{2ir}K_{ir} \right) \right] x + \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T P \left( C_{ir} + D_{ir}K_{ir} \right) \left( C_{ir} + D_{ir}K_{ir} \right)^T x + \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T P B_{ir}B_{ir}^T P x + \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T \left( C_{ir} + D_{ir}K_{ir} \right) \left( C_{ir} + D_{ir}K_{ir} \right)^T x
\]

The second term on the right-hand side of (11) is given as follows:

\[
\sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T \left( \frac{1}{\gamma} P B_{ir}B_{ir}^T P + \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \right) x = \left( \frac{1}{\gamma} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \right) x^T \left[ \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \right] x
\]

The last term on the right-hand side of (11) is given as follows:

\[
\sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T \left( \frac{1}{\gamma} P B_{ir}B_{ir}^T P \right) x = \left( \frac{1}{\gamma} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \right) x^T \left( \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \right) x
\]

\[
\sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \left( C_{ir} + D_{ir}K_{ir} \right) \left( C_{ir} + D_{ir}K_{ir} \right)^T x
\]

Upon noting inequality (10), we have

\[
x^T \left( A_{ir} + B_{2ir}K_{ir} \right)^T P + P \left( A_{ir} + B_{2ir}K_{ir} \right) x \leq \left( A_{ir} + B_{2ir}K_{ir} \right)^T P + P \left( A_{ir} + B_{2ir}K_{ir} \right)
\]

\[
\leq \frac{1}{\gamma^2} x^T P B_{ir}B_{ir}^T P + P \left( C_{ir} + D_{ir}K_{ir} \right) \left( C_{ir} + D_{ir}K_{ir} \right)^T x < 0
\]

When \( w_i = 0 \), with regard to (4), (11) and (14), we have

\[
\dot{V} = \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T \left( A_{ir} + B_{2ir}K_{ir} \right)^T P + P \left( A_{ir} + B_{2ir}K_{ir} \right) x < 0
\]

Thus, the closed-looped system (1) and (5) is asymptotically stable.

Combination of results (11), (12) and (13) gives rise to

\[
\dot{V} \leq \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} x^T Q_{ird} x - z^T z + \gamma^2 \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \left( w_i - \frac{1}{\gamma} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} P x \right)\frac{1}{\gamma^2} x^T P x
\]

Without loss of generality, we suppose \( x(0) = 0 \) and \( V(x(0)) = 0 \). Now, via integrating (16) for \( t \) running from 0 to \( \infty \), we obtain the following result:

\[
\|x(t)\|_2 \leq \|x(0)\|_2 - \lambda_{\max} \left( \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} Q_{ird} \right) \|x(t)\|_2 + \frac{1}{\gamma^2} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \left( w_i - \frac{1}{\gamma} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} P x \right)\frac{1}{\gamma^2} x^T P x
\]

where \( \lambda_{\max} \left( \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} Q_{ird} \right) \) denotes the maximal eigenvalue of matrix \( \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} \sum_{i=1}^{N} \eta_i \sum_{r=1}^{N} \eta_{ir} Q_{ird} \).

Thus the boundedness with an arbitrary constant \( \gamma \) under it is guaranteed, which ends stability investigation up an the proof.

IV. AN ILLUSTRATIVE EXAMPLE

To illustrate the design approach based on Theorem 1, we analyze the stability of a regulating system for room air-conditioning [19]. The state equation of the system is given as follows:

\[
\dot{T}_n = -\left( \frac{1}{T_1} + \frac{1}{T_2} \right) T_n - \frac{1}{T_1T_2} T_n + \frac{k_1k_2}{T_1T_2} u
\]

where \( T_n \) is the air temperature variable (°K) and \( \dot{T}_n \) is the rate of change of the air temperature variable (°K/minute) of the air-conditioned room. Coefficient \( T_1 \) is the time constant
of the air-condition room (min) and \( k_1 \) is the gain coefficient at the constant temperature room (°K/°K), whereas \( T_2 \) is the time constant of the steam heater (min) and \( k_2 \) is the gain coefficient of the electric actuator (°K/°K). Finally, \( u \) is the control variable.

When the temperature is at lower setup, \( T_1 = 20.30 \) \( T_2 = 1 \). And when the temperature is at higher setup, \( T_1 = 30.40 \) \( T_2 = 2.5 \).

In order to illustrate the stability of this system via switching control, upon appropriate coordinate transformation, we transform the problem into zero-equilibrium stability problem. Considering the redundancy circuit, we convert the common fuzzy model into the switched fuzzy model to arrive at scheduled temperature redundancy circuit, we convert the common fuzzy model into

\[
\begin{align*}
\mu_{R_{21}}(x_1) &= \mu_{R_{21}}^K(x_1) = 1 - \frac{1}{1 + e^{-2x_1}} , \\
\mu_{R_{21}}(x_1) &= \mu_{R_{21}}^K(x_1) = 1 - \frac{1}{1 + e^{-2x_1}} .
\end{align*}
\]

Now, assume \( \gamma = 1 \) in addition to choosing \( u_i(t) = \sum_{i=1}^{N_i} y_{ii} K_{ii} x(t) \), \( i = 1, 2 \). Then for the inequalities defined by Theorem 1 we obtain the following inequalities:

\[
\begin{align*}
&\sum_{i=1}^{N_i} y_{ii} \left[ (A_{\gamma i} + B_{2\gamma i} K_{iK_i})^T P + P (A_{\gamma i} + B_{2\gamma i} K_{iK_i}) \right] + \\
&\left[ (C_{\gamma i} + D_{\gamma i} K_{iK_i})^T \right] < 0,
\end{align*}
\]

\( i = 1, 2 \), \( j, \delta, p, q_i = 1, 2, \ldots, N_i \) . (17)

Further, we may choose \( \lambda_{ii} = 1 \). Then according to Schur Complement Lemma, upon a certain analysis, matrix inequalities (17) can be turned into an LMI problem that can be solved by using the LMI toolbox of Mathworks.

This way we obtain the controller gain matrices

\( K_{11} = [-0.131 \ -0.1148] \), \( K_{12} = [-0.0623 \ -2.302] \), \( K_{21} = [-4.9911 \ -2.4986] \), \( K_{22} = [-5.4991 \ -3.4986] \),

and also the positive definite matrix

\[
P = \begin{bmatrix}
0.0937 & 0.2146 \\
0.2146 & 0.6417
\end{bmatrix}.
\]

The design of the switching law is given by means of

\[
\sigma(x) = \arg \min \bigg\{ \left[ \frac{1}{x} \right] \bigg\} = \max \left\{ \lambda_{x \delta, p, q_i} \right\} \bigg\}
\]

\[
\begin{align*}
&\left[ (A_{\gamma i} + B_{2\gamma i} K_{iK_i})^T P + P (A_{\gamma i} + B_{2\gamma i} K_{iK_i}) \right] + \\
&\left[ (C_{\gamma i} + D_{\gamma i} K_{iK_i})^T \right] < 0, \\
&j, \delta, p, q_i = 1, 2, \ldots, N_i \}.
\end{align*}
\]

Then, the state feedback \( H_x \) robust control problem with setting up \( \gamma = 1 \) is solved.

The simulation results, which represent the performance of achieved in the closed-loop system operation with initial condition \( x(0) = [x_1(0) \ x_2(0)]^T = [-3 \ 0]^T \) and the designed control scheme are depicted in Figure 2. These clearly demonstrate a rapid convergence to the equilibrium state in a finite time and with acceptable transients in the time histories of the state variables.
The controlled state response of the closed-loop system with the PDC controller plus switching law according to Theorem 1.

| $\sigma(x) = \arg \min_{\lambda} \{ \gamma(x) \} = \max_{j, \theta, p, q} \{ $ |
| $\begin{bmatrix} (A_{ij} + B_{2ij}K_{ij})^T \gamma P + \gamma P(A_{ij} + B_{2ij}K_{ij}) \end{bmatrix} $ |
| $\begin{bmatrix} C_{ij} + D_{ij}K_{ij} \end{bmatrix}^T \gamma P B_{1ij} \left[ \begin{array}{c} x \end{array} \right] \} \quad x < 0 $ |
| $j, \theta, p, q \in \{1, 2\} \}$. |

Then, the state feedback $H_0$ robust control problem with setting up $\gamma = 1$ is solved.

The simulation results, which represent the performance of achieved in the closed-loop system operation with initial condition $[-3 0]^T$ and the designed control scheme are depicted in Figure 2. These clearly demonstrate a rapid convergence to the equilibrium state in a finite time and with acceptable transients in the state variables.

V. CONCLUSION

In this paper we have investigated the solving the problem of state feedback $H_0$ control for switched fuzzy systems, i.e. the sub-class of switched systems in which the subsystems are all represented by T-S fuzzy system models. In particular, considerable attention is focused on the representation model of switched fuzzy systems, which does not seem to have been considered deeply in previous works.

On the ground of the switching strategy, we derived a PDC fuzzy controller design and synthesis of switching law for the plant model in state-dependent form such that the $H_0$ control problem is solved. A sufficient condition for quadratic stability of these systems in closed-loop has been derived too. According to this condition it has been shown that we only need to check stability of a certain combination of sub-system matrices, which is easier to carry out, in order to establish the system stability.

Finally, a real-world example of room air-conditioning control system has been solved to illustrate the effectiveness of the proposed approach. Computer simulation experiment for an arbitrary initial state deviation form the equilibrium state has demonstrated a rapid convergence back to the equilibrium state in a finite time.

REFERENCES