Abstract – New stability synthesis for a class of affine nonlinear networked control systems within FMS plants is developed via the sampled-data approach. Such a networked control system consists of an affine nonlinear plant and a linear digital controller as well as short-range network induced time delays. It is viewed as a typical hybrid system model with network-induced delays. Sufficient conditions for stability synthesis of this class of affine nonlinear networked control systems are derived and the application illustrated by a simulated example.

1 INTRODUCTION
Nowadays, industrial computer control based technologies wherein configurations achieving closed-loop controlled performance through standard networks are found in fully automated robotic manufacturing systems such as FMS, and in particular in car industry and tele-robotic applications. Moreover, networked control systems appear to be rather typical for the modern automated manufacturing systems, and this paper gives some novel results for the case of FMS in which the impact of the network induced delays, albeit lower range, has to be observed.

The remarkable developments of communication and computer networks have enhanced their usage in control systems technology for complex hybrid and nonlinear plants, in particular the mechatronic ones and the remotely controlled robot movements, in order to achieve real-time requirements. The control systems remain feedback based essentially, however, the overall dynamics is essentially modified by the impact of network induced time delays both in direct-forward and feedback paths [1-10].

Figure 1 depicts an illustration of the typical setup of a Networked Control System (NCS), wherein the control loops are closed through a real-time network. In addition, Figure 2 presents the identification results for short-range network induced time delays from a set of empirical case study analysis and identification of typical network induced delays in the remote control of electrical drives [11]. In general, the impact of networks induced delays make the entire system dynamics become as the one of stochastic time-delay systems hence Lyapunov-Krasovskii theory is needed for deriving sufficient conditions for stochastic asymptotic stability of random delay systems [12]. Besides, this methodology involves Riccati equation solutions with a bound on the expected rate of change of the time delay [13].

The NCSs provide several rather distinct advantages in technological applications such as reduced system wiring, ease of system diagnosis, facilitated system maintenance, and increased systems agility. However, system operating safety and reliability are critically dependent on transferring signal data through networks timely and exactly. The limitation of network bandwidth has affect transmission timely and exactly so that the network-induced delay inevitably occurs; the characteristic features of delay phenomena depend on the types of network and the choice of hardware [4]. In turn, transmission delays may cause time-varying deterioration of the entire operation and only the performance of networked systems. For the delay, either constant or time varying, can degrade the performance of control systems and decrease the stability domain, and even make systems unstable [6].

Insofar the study of NCS has been mainly focused on stability analysis and stabilization of linear NCS, and there are available a number of results on these topics [2], [4].
However, very few results on nonlinear [15] networked control systems have appeared by now. In [1], a continuous model of nonlinear NCS is used, and the network-induced delay is described as error state-vector. Then the error is considered as a perturbation of the original plant, and a condition for stability of the closed-loop systems is given. In [5], the results on input-output stability of nonlinear NCS are presented for a large class of network scheduling protocols.

This paper presents a study on the stability of a class of networked, affine-nonlinear, control systems for FMS mechatronic applications by using the sampled-data control systems approach. The adopted system model is a nonlinear generalization of the linear model in [4]. A state feedback is provided by a digital controller along with sufficient conditions for stability form the proposed synthesis design.

II NETWORKED CONTROL SYSTEMS: TIME DELAYS, ASSUMPTIONS, AND A PREVIOUS RESULT

Networked control systems, such as illustrated in Figure 1, that are distributed on a short range area within a couple of hundred kilometers do cause network induced time delays. The main features of this time delay may be inferred from Figure 2 a, b [11]. It is the impact of this delay and these NCSs that are investigated in this paper with respect to control design synthesis and stability property.

On the grounds of the given empirical findings, the below stated assumptions are adopted in here, albeit they are which are commonly used in the literature. It should be further noted, the case with network-induced delay which is less than one sampling period, \( \tau_k < h \), which is typical for controlled and robots employed within FMS plants is considered.

Assumption 1 [4]. The network-induced delay consists of sensors-to-controller delay \( \tau_{sc,k} \) and controller-to-actuator delay \( \tau_{ca,k} \). For fixed control law (time-invariant controller), sensor-to-controller delay and controller-to-actuator delay can be lumped together as \( \tau_k = \tau_{sc,k} + \tau_{ca,k} \) for analysis purposes [4].

Assumption 2 [4]. The system architecture studied is characterized with: (1) clock-driven sensors that sample the plant outputs periodically at sampling instants; (2) an event-driven controller which can be implemented by an external event interrupt mechanism and which calculates the control signal as soon as the sensor data arrive; and (3) event-driven actuator, which means the plant inputs are changed as soon as the data become available [4].

Therefore the considered model of the affine nonlinear NCS with network-induced delay can be described by means of the following equations:

\[
\dot{x}(t) = f(x(t)) + g(x(t))u(t),
\]

\[ t \in [kh + \tau_k, (k + 1)h + \tau_{k+1}], \quad (1a) \]

\[
u(t') = -Kx(t - \tau_k), \quad t' \in [kh + \tau_k, k = 0, 1, 2, ...].
\]

\[ (1b) \]

Respectively, the symbols denote: state \( x(t) \in \mathbb{R}^n \), control \( u(t) \in \mathbb{R}^m \), and output \( y(t) \in \mathbb{R}^p \) vectors whereas \( K \) has compatible dimension. Function \( f(x) \) is continuously differentiable, and \( g(x) \in \mathbb{R}^{nxm} \). The subscript \( k \) represents the sampling instant; the sampling period is \( h \). The sampling signal coming from zero-order holder is the input of plant hence \( u(t') \) is the piecewise constant continuous signal and changes value at \( kh + \tau_k \) only. It is important to note, however, that system (1) can also be represented by means of the following model:

\[
\dot{x}(t) = Ax(t) + Bu(t) + f_1(x(t)) + g_1(x(t))u(t),
\]

\[ t \in [kh + \tau_k, (k + 1)h + \tau_{k+1}], \quad (2a) \]

\[
u(t') = -Kx(t - \tau_k), \quad t' \in [kh + \tau_k, k = 0, 1, 2, ...].
\]

\[ (2b) \]

In here, \( A \in \mathbb{R}^{nxn} \) represents Jacobian matrix of \( f(x) \) at point \( x = 0 \), i.e. \( A = \left[ \frac{\partial f}{\partial x} (0) \right]_{n \times n} \), and \( B \in \mathbb{R}^{nxm} \) denotes the value of \( g(x) \) in Eq. (1) at point \( x = 0 \), i.e. \( B = g(0) \), and \( f_1(x) \), \( g_1(x) \) represent the nonlinear terms. Furthermore

\[
f(x) = Ax + f_1(x)
\]

(3)
where $A \in \mathbb{R}^{n \times n}$ since $f \in C^1[\mathbb{R}^n, \mathbb{R}^n]$ and $f(0) = 0$. Also
\[
g(x) = B + g_1(x)
\] exists where $g(0) = B$ and $B \in \mathbb{R}^{n \times m}$. The condition for the stability of linear NCS has been found \cite{4}, \cite{14}, and it is given here by Lemma 2.1.

**Lemma 2.1.** The equilibrium $(x^T, u^T)^T = (0^T, 0^T)^T$ of the linear NCS
\[
\dot{x}(t) = A x(t) + B u(t),
\]
\[
t \in [k h + \tau_k, (k + 1)h + \tau_{k+1}],
\] and
\[
u(t^+) = -K x(t - \tau_k), t \in \{kh + \tau_k, \tau_{k} \mid k = 0, 1, 2, \ldots\}
\]
is exponentially stable if and only if the matrix
\[
H = \begin{bmatrix}
\Phi - \Gamma_0(\tau_k) K & \Gamma_1(\tau_k) \\
-K & 0
\end{bmatrix}
\]
is Schur stable.

In the sequel, further consideration is carried out split into two cases. One is when both $f_1(x)$ and $g_1(x)$ are vanishing perturbations, and the other is when they are not.

For the first case, with $f_1(0) = 0$ and $g_1(0) = 0$, the next lemma is a novel result the proof of which is given in the appendix. Clearly, $(x^T, u^T)^T = (0^T, 0^T)^T$ is an equilibrium of (1).

**Lemma 3.1:** For any $k \in \mathbb{Z}^+$ and any given $\mu > 0$, there exist $\gamma' = \gamma'(\mu) > 0$ and $\delta_0 = \delta_0(\mu) > 0$ such that when
\[
\max \{\|f_1(x)\|, \|g_1(x)\|\} \leq \gamma' \|x\|
\]
is satisfied, the inequality
\[
\|\Delta(kh)\| \leq \int_{kh}^{kh+h} e^{Lh} \|G(x(s))\| ds < \mu \|w(kh)\|
\]
holds whenever $\|w(kh)\| < \delta_0$, where
\[
G(x(s)) = f_1(x(s)) + g_1(x(s))u(s).
\]
The proof has been dropped out due to paper size limits though it was available to the anonymous reviewers. The subsequent theorem establishes a condition for stability of the original nonlinear system (1).

**Theorem 3.1.** The equilibrium $(x^T, u^T)^T = (0^T, 0^T)^T$ of the affine nonlinear NCS (1) is uniformly asymptotically stable if the equilibrium $(x^T, u^T)^T = (0^T, 0^T)^T$ of the respective linear NCS is exponentially stable, or equivalently, if the matrix $H$ of the linear part is Schur stable.

**Proof:** The solution of (2) is given by
\[
x(t) = e^{A(t-kh)} x(kh) + \int_{kh}^{t} e^{A(t-s)} B u(s) ds + \int_{kh}^{t} \int_{kh}^{s} e^{A(t-s)} [f_1(x(s)) + g_1(x(s))u(s)] ds ds.
\]
for all $t \in [kh, kh + h]$. In particular, at $t = kh + h$, if any $\tau_{kh} < h$, it follows:
\[
x(kh + h) = e^{Ah} x(kh) + \int_{kh}^{h} e^{As} ds Bu(kh) + \int_{kh}^{h} e^{Ah} ds Bu(kh - h) + \int_{kh}^{kh+h} e^{A(hkh-h)} [f_1(x(s)) + g_1(x(s))u(s)] ds.
\]
Upon defining $w(kh) = \begin{bmatrix} x(kh) \\ u(kh - h) \end{bmatrix}$ the augmented state vector, for the closed-loop system one obtains
\[
w(kh + h) = Hw(kh) + \begin{bmatrix} \Delta(kh) \\ 0 \end{bmatrix}
\]
where
\[
\Delta(kh) = \int_{kh}^{kh+h} e^{A(hkh-h)} [f_1(x(s)) + g_1(x(s))u(s)] ds
\]
and the augmented state matrix $H$ is given by (6).

Equation (10-b) represents the nonlinear NCS (1) at discrete instants in time $k = 1, 2, \ldots$. To show that the trivial solution of (1) is uniformly asymptotically stable, it is necessary to show that the trivial solution of (11) is also sufficient. In what follows, the trivial solution of (11) is proven to be uniformly asymptotically stable.

Since by assumption $H$ is Schur stable, there exists a positive definite symmetric matrix $P$ such that $H^T PH - P = -I$, where $I \in \mathbb{R}^{(n+m)(n+m)}$ denotes the respective identity matrix. A candidate Lyapunov function is defined as
\[
V(w(kh)) = w^T(kh) P w(kh)
\]
where $w \in \mathbb{R}^{n+m}$. By letting $\Omega(kh) = [\Delta^T(kh), 0^T]^T$,
Eq. (11) is written more concisely as
\[ w(kh + h) = Hw(kh) + \Omega(kh) \]  
(13)
The first forward difference of (12) along the solutions of (13) yields:
\[
\Delta V(w(kh)) = V(w(kh + k) - V(w(kh))
= w^T(kh) [H^T \Phi - P] w(kh) + 
+ 2\Omega^T(kh) PH w(kh) + \Omega^T(kh) \Omega(kh)
\leq -\|w(kh)\|^2 + 2\|\Delta(kh)\|\|PH\|\|w(kh)\|
+ \|\Delta(kh)\|^2 \|P\|. 
\]  
(14)
From Lemma 3.1 the following can be inferred: If it were chosen a \( \mu_0 > 0 \) such that \( 2\mu_0 \|PH\| + \mu_0^2 \|P\| < 1 \), there exists a \( \delta_0(\mu_0) > 0 \) such that:
\[
\Delta V(w(kh)) < -\|w(kh)\|^2 = 
+ 2\mu_0 \|PH\| \|w(kh)\| + \mu_0^2 \|P\| \|w(kh)\|
\]  
(15)
wherever \( \|w(kh)\| < \delta_0(\mu_0) \). From (12) and (15) It follows that
\[
\lambda_{\min}(P)\|w(kh + h)\|^2 
\leq V(w(kh + h)) < V(w(kh)) 
\leq \lambda_{\max}(P)\|w(kh)\|^2 
\]  
(16)
where \( \lambda_{\min}(P) \) and \( \lambda_{\max}(P) \) denote the smallest and largest eigenvalues of \( P \), respectively. Further let it be \( d = (\lambda_{\min}(P) / \lambda_{\max}(P))^{1/2} \delta(\mu_0). \) If for some \( k_0 \in Z^+ \) holds \( \|w(k_0h)\| < d \), inequality (16) yields \( \|w(k_0h + h)\| < \delta_0(\mu_0) \). Then if (14) is applied for \( k = k_0 + 1 \), it yields
\[
V(w((k_0 + 2)h)) < V(w((k_0 + 1)h)) < V(w(k_0h)).
\]
Next, replacing \( k_0 + 1 \) in (16) by \( k_0 + 2 \) yields \( \|w((k_0 + 2)h)\| < \delta_0(\mu_0) \). Hence (15) is satisfied for \( k > k_0 \) whenever \( \|w(k_0h)\| < d \). Therefore, the trivial solution of (11) is uniformly asymptotically stable in the region \( B_d = \{w \in R^{(s+1)}: \|w\| < d \} \) of \( w = 0 \).

In the next step, it is explained that the trivial solution of (11) is uniformly asymptotically stable whenever if the solution of (11) is uniformly asymptotically stable. Since \( u(t) \) is piece-wise constant and changes its values at instants \( kh + \tau, k = 0, 1, 2, \ldots, \) for \( t \in [kh, (k + 1)h) \), from (13) it follows:
\[
\|x(t)\| \leq e^{H\Phi \gamma} \|x(\tau)\| + 
he^{H\Phi \gamma} \|B\| \|x(\tau)\| + \|w(kh - h)\| + 
+ \int^h_{kh} e^{H\Phi \gamma \gamma} \|G(x(s))\| ds 
\]  
\[ < \|x(\tau)\| + \|w(kh - h)\| + 
\]  
(17)
Thus whenever \( \|w(k_0h)\| < d \) (noting \( d \) is independent of \( k_0 \)) for \( t \in [kh, kh + h] \), state system \( x(t) \) converges to the origin simultaneously with the augmented \( w(kh) = \left[ x^T(kh), u^T((k - 1)h) \right]^T \). It is hence concluded that the trivial solution of the original system (1) is uniformly asymptotically stable if the solution of (11) is uniformly asymptotically stable, and this completes the proof.

**Remark 1:** Here system (1) is dealt with as a hybrid dynamic system, which involves simultaneously in some parts of the system discrete-time dynamics and in other parts continuous-time dynamics. Therefore, the proof of Theorem 3.1 does not follow directly from Lyapunov’s First Method applied.

**Remark 2:** Theorem 3.1 constitutes local result and estimate of an attraction region is a challenging issue. This will be shown further below by an illustrative example.

**Remark 3:** When System (1) degenerates into a linear NCS, i.e. \( f_1(x) = 0 \) and \( g_1(x) = 0 \), then Theorem 3.1 coincides with the result of Zhang et al. (2001). Thus, Theorem 3.1 is a nonlinear generalization of their result.

For the second case of non-vanishing perturbations, either \( f_1(0) = 0 \), or both does not hold, which implies that \( (x^T, u^T) = (0^T, 0^T) \) is no longer an equilibrium of system (1). Hence no local stability can be expected. It will be shown in the sequel, however, that still it is possible to make \( w(kh) \) ultimately bounded provided the perturbation terms \( f_1(x) \) and \( g_1(x) \) are sufficiently small.

**Lemma 3.2.** For any \( k \in Z^+ = \{0, 1, 2, \ldots\} \) and any given \( \mu > 0 \), there exist \( \gamma_2 = \gamma_2(\mu) > 0 \) and \( \delta_0 = \delta_0(\mu) > 0 \) such that whenever
\[
\max \{\|f_1(x)\|, \|g_1(x)\|\} \leq \gamma_2 \]  
(18)
is satisfied, for \( t \in [kh, kh + h] \) the following inequality
\[ \|\Delta(kh)\| \leq \int_{kh}^{kh+h} e^{t \Phi} \|G(x(s))\| ds < \mu \|w(kh)\| + J\gamma_2 \]  
(19)

with \( G(x(s)) = f_1(x(s)) + g_1(x(s))u(s) \),  
\( J = he^A44 \), and  
\[ c_1 = e^H f_3 \sqrt{1 + \tau_k^2 (\|B\| + \gamma_2)^2} \],
holds wherever \( c_1 \|w(kh)\| + \gamma_2 \tau_k < \delta_0 \).

The proof is similar to that of Lemma 3.1 hence omitted.

Finally, from Lemma 2.1 and Lemma 3.2 the next theorem is straightforward.

**Theorem 3.2.** The affine nonlinear NCS (1) is uniformly bounded if the equilibrium \((x(0),0,0)\) of the linear NCS is exponentially stable, or equivalently, if the matrix \( H \) is Schur stable.

**Remark 4:** Similar results as above can be established in the presence of combined perturbations provided

\[
\max \|f_1(x)\|,\|g_1(x)\| \leq \gamma_1 \|x\| + \gamma_2, \tag{20}
\]
is satisfied.

**IV ILLUSTRATIVE EXAMPLE**

Consider now the following affine nonlinear system that is network controlled:

\[
\dot{x}(t) = -\sin x(t) + (1 + x(t) + x^2(t))u(t), \quad t \in [kh + \tau_k, (k+1)h + \tau_{k+1}], \tag{21a}
\]

\[
u(t^*) = -Kx(t - \tau_k), \quad t \in [kh + \tau_k, k = 0,1,2,\ldots]. \tag{21b}
\]

The plant model comprising the linearized part is given as follows:

\[
\dot{x}(t) = x(t) + u(t) + (-\sin x(t) - x(t)) + 
(x(t) + x^2(t))u(t), t \in [kh + \tau_k, (k+1)h + \tau_{k+1}],
\]

\[
u(t^*) = -Kx(t - \tau_k), k = 0,1,2,\ldots.
\]

where the linear part is the same as the scalar hybrid NCS [4]. It is obvious that \( \gamma_1 = 1 \), \( A = \frac{\partial f}{\partial x}(0) = 1 \), \( g(0) = B = 1 \), \( \mu_1 = 1 \).

Let \( K = 2 \) be given from some previous study, and let \( x(0) = 1.5 \) be the initial state of the system for the purpose of simulation. Next, the \( w(kh) \) is defined as equation (10). For sampling period \( h = 0.2 \) s, one can find the bound of the delay \( 0 < \tau_k < 0.526 s \) if \( H \) is Schur stable. For \( \tau_k < h \), let \( \tau_k = 0.18 s \), then

\[
H = \begin{bmatrix}
1.1810 & 0.2012 \\
2.00 & 0.00
\end{bmatrix}
\]
is Schur stable, and moreover for the matrix \( P \) one can establish:

\[
P = \begin{bmatrix}
4.2690 & -7.19 \\
-7.19 & 18.0759
\end{bmatrix}
\]

Further, by choosing \( \mu_0 = 0.01024 \), one ensures the condition \( 2\mu_0 \|PH\| + \mu_0^2 \|P\| < 1 \) is fulfilled. And should it be chosen \( \mu_0 = 0.01024 = \mu \), one gets the other parameters \( \delta_0(\mu_0) = 0.027035 \) and \( d = 0.0065 \) needed.

![Figure 3](image3.png)

The state response of the example system.

![Figure 4](image4.png)

The control input of the example system with the network induced time delay.

The respective simulation investigation was carried out by using Matlab and Simulink. The respective computer results on the plant’s state and control input are depicted in Figure 3 and Figure 4. The system state \( x(t) \) is
guaranteed to converge to the origin simultaneously along with $w(kh)$ whenever $\|\omega(k_h)\| \leq d$.

V CONCLUDING REMARKS

For networked control systems with a class of affine nonlinear plants sufficient conditions for stability synthesis of the operating equilibrium in presence of network induced delay that are typical for motion control with FMS plants is proposed in here. It has been derived via the sampled-data systems approach. Further, from the point of view of the theory of hybrid systems, the interaction between a continuous-time affine nonlinear plant and a digital controller of state feedback has been analyzed. A potential extension of these results is to handle the situation in cases when varying sampling period is employed, whereas the technique for choosing the gain is the topic of a future paper.

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