Fuzzy Direct Adaptive Sliding Mode Control of Interconnected Large-Scale Systems

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Abstract— Fuzzy direct adaptive sliding mode control design for a class of interconnected large-scale non-linear systems with unknown functions has been resolved. It makes use of the fuzzy control method via fuzzy logic approximation with fuzzy sliding mode control to approximate the ideal controller synthesised. Fuzzy sliding mode control is introduced to attenuate the fuzzy approximation errors. The (sub)system parameters are being regulated on-line by adaptive laws and stabilised by virtue of Lyapunov's second method applied.

Index Terms: Control design; fuzzy direct adaptive control; interconnected large-scale systems; sliding mode control; stability.

I. INTRODUCTION

TERM fuzzy direct adaptive control (FDAC) is referred to control system architecture in which a fuzzy logic system is employed as controller [10], [11]. These direct adaptive controllers are capable of utilizing directly the fuzzy control rules, but not the fuzzy description information. Fuzzy adaptive control of nonlinear unknown dynamic systems, fuzzy direct adaptive control concept [1], [6], [7] has attracted as much attention of many researchers as the indirect one [4], [5], [8]. However, most of the existing results are about control of systems without interconnections with uncertainty structure, i.e. common cases of large-scale or non-linear systems. Tong, Chai and Li [6], [7] have found that adding an adaptive parameter and using the fuzzy sliding mode control could attenuate the impact of the interconnected terms provided the interconnections satisfied certain strict conditions. Apparently, in potential applications, the designed controller has considerable limitations because of the constriction conditions on the interconnections. Hence a new issue has arisen: whether a new adaptive controller can be synthesized to realize the control objective when the interconnections are unknown and do not satisfy these restrictive conditions, or the conditions are relaxed? It is shown in this paper that the answer is confirmative.

A feasible design synthesis of a new fuzzy direct adaptive controller with guaranteed stability for interconnected non-linear systems with unknown or uncertainty interconnections is presented. For reasons of dimensionality of the controller, the second kind of FDAC [10], [11] has been adopted. It should be noted that an alternative design synthesis for the same class of systems possessing similarity is feasible via making use of neural networks as shown in [12]. The present work is part of authors' research endeavours to explore new ways in controlling complex systems via compatible combining math-analytical and soft-computing techniques.

The paper is written as follows. Section II introduces the system model, control objective and basic assumptions. Section III presents an outline of design synthesis and the specific form of the controller. The main theoretical result on stability and its proof are given in Section IV. In Section V the conclusions are drawn and future research indicated. This design and its main result have been applied to adequate examples including double-inverted pendulum system, and the respective simulation results obtained seem to be almost ideal and rather promising. Example, however, had to be omitted due to space limits.

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II. PLANT SYSTEM CLASS, CONTROL OBJECTIVE, AND BASIC ASSUMPTIONS

Consider the class of interconnected non-linear systems with $N$ subsystems that is represented by means of the following model equations:

$$
\sum_{i} y_i^{(n)} = a_i(x_i) + b_i(x_i)u + d_i(X,t), \quad i = 1,2,\ldots,N.
$$

(1)

Here $x_i \in \mathbb{R}^n$ is the state vector of subsystem $i$, $u$ is the input, and $y_i$ is the output. Functions $a_i(x_i), b_i(x_i) \in \mathbb{R}^n$ represent the unknown dynamics (i.e., unknown functions), and function $d_i(X,t) \in \mathbb{R}$ represent the unknown or uncertainty interconnections between subsystems. These all are assumed to be smooth functions, and also $x = (x_1,\ldots,x_N)$.

The control objective can be stated as follows:

Design direct adaptive fuzzy controls $u_i(x_i | \theta_i)$ for the case when both the unknown dynamics of the subsystems and also of the interconnections exist. The controller should satisfy these requirements:

(1) The overall system is stable in the sense of Lyapunov, i.e., all the system variables are bounded to tolerable level.

(2) The tracking error of each subsystem asymptotically converges to zero or to a desired neighborhood of zero.

Then the following realistic assumption is adopted.

**Assumption 1:** The class of systems (1) is assumed to satisfy:

(i) $|a_i(x_i)| \leq M_{i0}(x_i)$;

(ii) $c_i \leq M_{i1}(x_i) \leq b_i(x_i) \leq M_{i2}(x_i)$;

(iii) $|b_i^{-1}(x_i)d_i| = |\Delta b_i^{-1}(x_i)x_i| \leq M_{i3}(x_i)$;

where $M_{ij}(x_i)$ are known functions, and $c_i$ are known constants.

Let $y_{im}$ be the reference output desired. Assume $y_{im}, \hat{y}_{im}, \ldots, y_{in}^{(n)}$ are all bounded and measurable. Let denote $x_{im} = (y_{im}, \hat{y}_{im}, \ldots, y_{in}^{(n-1)})$ and $x_i = (y_i, \hat{y}_i, \ldots, y_i^{(n-1)})$, and define the tracking error of subsystem $i$ as $e_{i0} = y_{im} - y_i$. Also, let define $e_i = (e_{i0}, \hat{e}_{i0}, \ldots, e_{i0}^{(n-1)})^T$ and $K_i = (k_{i0}, \ldots, k_{i0})^T$, with $K_i$ selected to satisfy Hurwitz type $\hat{L}_i(s) = \frac{s^{n}}{s^{n} + k_{i(n-1)}s^{(n-1)} + \cdots + k_{i0}}$ having roots in the open left half-plane $s$.

Next, let define the sliding hyper planes $s_i(t)$ as

$$
s_i(t) = k_{i0}\hat{e}_i + k_{i1}\hat{\hat{e}}_i + \cdots + k_{(i(n-1)}\hat{e}_{i(n-2)} + e_i^{(n-1)}.
$$

(2)

Because $a_i(x_i), b_i(x_i)$ are unknown and $d_i \neq 0$, a fuzzy logic system can be utilized to approximate the controls $u_i^*$, where

$$
u_i^* = \frac{1}{b_i(x_i)}(-a_i(x_i) - d_i(X,t) + \eta_is_{im}(t) + \hat{s}_i(t) + e_i^{(n)} - e_i^{(n)})
$$

(3)

and $\eta_i > 0$, $s_{im} = s_i + \phi_i\text{sat}(s_i | \phi_i)$, $\phi_i > 0$.

Let define bounded closed sets $A_{id}, A_i$ as

$$A_{id} = \{x_i | \|x_i - x_{im}\|_{p,z} \leq 1\},$$

$$A_i = \{x_i | \|x_i - x_{im}\|_{p,z} \leq 1 + \phi_i\},$$

$$\|x_i\|_{p,z} = (\sum_{i=1}^{N} x_{i0}^{p,i})^{1/p}. \quad \text{in here, } \phi_i \text{ is the width of the transition area, } x_{im} \text{ is a fixed point in } \mathbb{R}^N, \text{ and } \|x_i\|_{p,z} \text{ is a kind of } p\text{-norm, and } \{\pi_{i}^{n}\}^{n}_{i=1} \text{ is the given power weight. For arbitrary given } \varepsilon_i > 0 \text{, on the grounds of approximation theory, there exists fuzzy logic system }$$

$$u_i(X | \theta_i) = \theta_i^T \hat{\xi}_i(X) \text{ such that for } X \in \bigcap_{i=1}^{N} A_i \text{ the approximation error bounded}$$

$$\|u_i^* - u_i(X | \theta_i)\| \leq \varepsilon_i. \quad (4)$$

Then the decentralized fuzzy controller can be designed as

$$u_i = (1 - m_i(t))u_{ad} - m_i(t)k_{i1}(s_i,t)u_{fs} - k_{i2}(s_i,t)u_{fs}, \quad (5a)$$

where $u_{ad} = u_{i}(X | \hat{\theta}_i) - \hat{\varepsilon}_i u_{fs}$ is the adaptive part, $u_{fs}$ is the fuzzy sliding mode controller, $k_{i1}(s_i,t) > 0$, $k_{i2}(s_i,t) > 0$, and $m_i(t)$ is a mode transformation function possessing property $0 \leq m_i(t) \leq 1$. The latter is defined as follows:

$$m_i(t) = \max\{0, \text{sat}\left(\frac{|x_i - x_{im}|_{p,z}}{\phi_i}\right)\}. \quad (5b)$$

And, this completes all the essential preliminaries.
III. ON THE DESIGN SYNTHESIS

Following the recent theoretical developments in fuzzy control (e.g. see [1], [9], [10]), let suppose that fuzzy linguistic description rules of the unknown function \( u_j(x_j) \) have the form \( R_j^{(m)} \):

IF \( x_{11} = M_{11}^m \) and \( \cdots \), and \( x_{1n} = M_{1n}^m \),
and \( x_{21} = M_{21}^m \) and \( \cdots \), and \( x_{2n} = M_{2n}^m \),
\( \cdots \),
and \( x_{N1} = M_{N1}^m \) and \( \cdots \), and \( x_{Nn} = M_{Nn}^m \),
THEN \( u_j(x) = E_j^m \).

In here, \( M_{kj}^m \), \( E_{ij}^m \) and \( (k = \{1, \ldots, n\} \) are the fuzzy sets on \( R \), their membership functions are chosen to be of Gaussian form

\[
u_{ij}(x) = \frac{\sum_{m=1}^{p} \sum_{l=1}^{N} \prod_{j=1}^{n} \exp\left[\frac{-(x_{ij} - \bar{x}_{ij,m})^2}{\sigma_{ij,m}^2}\right]}{\sum_{m=1}^{p} \sum_{l=1}^{N} \prod_{j=1}^{n} \exp\left[\frac{-(x_{ij} - \bar{x}_{ij,m})^2}{\sigma_{ij,m}^2}\right]},
\]

where \( \theta_j \) represents the set of the adjustable parameters \( \bar{x}_{ij,m}^j, \bar{x}_{ij,m} \) and \( \sigma_{ij,m}^j \).

By making use of Taylor formula, the following formula can be derived

\[
u_{ij}(X|\theta_j) - \nu_{ij}(X|\hat{\theta}_j) = \Phi_j \left( \frac{\partial \nu_{ij}(X|\hat{\theta}_j)}{\partial \theta_j} \right) + O(\Phi_j^2)
\]

where \( \Phi_j = \theta_j - \hat{\theta}_j \) by definition. Then, to calculate \( \frac{\partial \nu_{ij}(X|\hat{\theta}_j)}{\partial \theta_j} \), let adopt the following algorithm

\[
\frac{\partial \nu_{ij}}{\partial \bar{x}_{ij,m}} = \sum_{i=1}^{b_{ij}} b_{ij}^m, \\
\frac{\partial \nu_{ij}}{\partial \bar{x}_{ij}} = \frac{(\bar{x}_{ij} - \hat{u}_j)b_{ij}}{\sum_{i=1}^{b_{ij}} b_{ij}^m} - \frac{2(x_{ij} - \bar{x}_{ij,m})}{(\sigma_{ij,m}^2)} ,
\]

\[
\frac{\partial \nu_{ij}}{\partial \sigma_{ij,m}} = \frac{2(x_{ij} - \bar{x}_{ij,m})}{(\sigma_{ij,m}^2)},
\]

where \( b_{ij}^m = \prod_{f=1}^{n} \exp\left(-\frac{(X_{ij} - \bar{x}_{ij,f})^2}{\sigma_{ij,f}^2}\right) \). The above formulas represent the derivative of \( u_j \) in Eq. (6).

The fuzzy sliding mode controller [3], the \( u_{j_{fi}} \) above, can be designed via the procedure presented below.

Firstly, define the linguistic description of \( s_i \) and \( u_{j_{fi}} \) as follows:

\[
T(s_i) = \{NB, NM, ZR, PM, PB\} \\
= \{C_{1i}, C_{2i}, C_{3i}, C_{4i}, C_{5i}\}.
\]

\[
T(u_{j_{fi}}) = \{NB, NM, ZR, PM, PB\} \\
= \{F_{1i}, F_{2i}, F_{3i}, F_{4i}, F_{5i}\}.
\]

Here “NB”, “NM”, “ZR”, “PM”, “PB” are the standard set labels in fuzzy control. They have been taken to be triangle-shaped fuzzy sets, and Figure 1 depicts the membership functions; alternatively, if preferred these can be bell-shaped.

![Fig.1. The fuzzy membership functions adopted](image)

Secondly, using the intuitive inference methodology, the fuzzy relationship can be built between the tracking error \( e_{fi} \) and the fuzzy controls \( u_{j_{fi}} \) as \( R_j^i \):

IF \( e_{fi} \) is \( C_i^{f_{1}} \), THEN \( u_{j_{fi}} \) is \( B_{j_{1}}^{e_{ji}} \) \( (j = 1, \ldots, 5) \)
input \( e_{fi} \) is \( C_i \), output \( u_{j_{fi}} \) is \( F_{ji} \).

From the \( j \)-th rule, it can be derived that the underlying fuzzy relation is \( R_j^i = C_i^{f_{1}} \times F_{ji}^{e_{ji}} \), that is

\[
R_j^i (e_{fi}, u_{j_{fi}}) = C_i^{f_{1}}(e_{fi}) \cap F_{ji}^{e_{ji}}(u_{j_{fi}}).
\]
Therefore, the overall fuzzy relation \( R_i = \bigcup_{j=1}^{5} R_i^j \) is given as follows:

\[
R_i^j (e_u, u_{\mu_i}) = \bigcup_j \left[ C_i^j (e_u) \cap F_i^{\mu_i-j} (u_{\mu_i}) \right].
\]

For a given input fuzzy set \( C_i^j \), the output fuzzy set \( F_i^{\mu_i-j} \) can be calculated by means of singleton, max-min fuzzy reasoning:

\[
F_i (u_{\mu_i}) = \bigcup_j \left[ C_i^j (e_u) \cap F_i^{\mu_i-j} (u_{\mu_i}) \right].
\]

Then by employing centre-average defuzzifier, the specific control output is obtained via the formula:

\[
u_{\mu_i} = \frac{\int_{\mu_i} x F_i (u_{\mu_i}) du_{\mu_i}}{\int_{\mu_i} F_i (u_{\mu_i}) du_{\mu_i}}.
\]

By defining \( x_i = \frac{e_i}{\phi} \) when \( |e_i| \geq \phi \), the mathematical expression of this formula (see [2], [3]) can be derived as

\[
u_{\mu_i} = \left\{ \begin{array}{ll}
-1 & x_i < -1, \\
\frac{2x_i + 3}{2} & 1 \leq x_i < -0.5, \\
\frac{x_i(2x_i + 1)}{4x_i^2 + 6x_i + 1} & -0.5 \leq x_i < 0, \\
\frac{2x_i + 2x_i - 1}{4x_i^2 + 2x_i - 3} & 0 \leq x_i < 0.5, \\
\frac{2x_i^2 - 2x_i - 1}{2x_i^2 - 2x_i + 1} & 0.5 \leq x_i < 1, \\
1 & x_i \geq 1.
\end{array} \right.
\]

It is easy to check that \( u_{\mu_i} = \text{sgn}\left(\frac{e_i}{\phi}\right) \).

And, thirdly, the use of Eqs. (3) and (5) yields:

\[
\dot{s}_i(t) + \eta_s \Delta \dot{s}_i(t) = b_i(x_i)[u_i^* - u_i] - \eta_i \varphi_i \text{sat}(\frac{s_i}{\varphi_i})
\]

\[
= b_i(t)(1 - m_i(t))[u_i^* - u_i(X|\hat{\theta}_i)] + \hat{\varepsilon}_i u_{\mu_i}]
\]

\[
m_i b_i[u_i^* + k_i u_{\mu_i}] + b_i k_2 u_{\mu_i} - \eta_i \varphi_i \text{sat}(\frac{s_i}{\varphi_i})
\]

\[
= b_i(t)(1 - m_i(t))\Phi_i \frac{\partial \hat{\theta}_i}{\partial \hat{\theta}_i} + O(\|\Phi_i\|^2)
\]

+ \dot{\theta}_i \left(1 - m_i(t)\right)

\[
+ b_i(t)(1 - m_i(t))[u_i^* - u_i(X|\hat{\theta}_i)] + \hat{\varepsilon}_i u_{\mu_i}]
\]

\[
n m_i b_i[u_i^* + k_i u_{\mu_i}] + b_i k_2 u_{\mu_i} - \eta_i \varphi_i \text{sat}(\frac{s_i}{\varphi_i})
\]

\[
(7)
\]

Practical stability considerations of this design synthesis require an additional, but realistic, assumption to be associated with the class of interconnected systems (1).

**Assumption 2:** For system class (1) the following assumptions also hold:

\[
(iv) \quad O(\|\Phi_i\|^2) \leq M_4 (x_i),
\]

\[
(v) \quad |d_i(X,t)| \leq M_5 (X).
\]

Then in the proposed design, the following quantities are chosen:

\[
k_{ii} (s_{ii}, t) = \frac{1}{M_{ii}(x_i)}[M_{ii}(x_i) + M_{ii}(X) +
\]

\[
+ M_{ij} + \eta_{ii} d_{ii}(t) + s_{ii}(t) - e_i^{(n_i)} + x_i^{(n_i)}]\n\]

\[
k_{ii} (s_{ii}, t) = \frac{1}{M_{ii}(x_i)}[M_{ii}(t) + M_{ij} \cdot]
\]

The adopted parameter adaptive laws are constructed by means of the rule and formulae given in the sequel.

Namely, when \( \sigma_{ii}^m = \sigma \), adopt and apply

\[
\sigma_{ii}^m = -\eta_{ii} (1 - m_i(t)) s_{ii} \frac{\partial \hat{\theta}_i}{\partial \sigma_{ii}}
\]

\[
\text{if } \eta_{ii} (1 - m_i(t)) s_{ii} \frac{\partial \hat{\theta}_i}{\partial \sigma_{ii}} < 0, (8a)
\]

\[
\sigma_{ii}^m = 0 \quad \text{if } \eta_{ii} (1 - m_i(t)) s_{ii} \frac{\partial \hat{\theta}_i}{\partial \sigma_{ii}} \geq 0.
\]

Or if \( |\dot{\theta}_i| < M_{\rho} \)

\[
\dot{\theta}_i = -\eta_{ii} (1 - m_i(t)) s_{ii} \frac{\partial \hat{\theta}_i}{\partial \hat{\theta}_i}
\]

\[
\text{if } |\dot{\theta}_i| < M_{\rho} \text{ and } s_{ii} \theta_i \frac{\partial \hat{\theta}_i}{\partial \hat{\theta}_i} \geq 0, (9a)
\]

\[
\dot{\theta}_i = P_1 (-\eta_{ii} (1 - m_i(t)) s_{ii} \frac{\partial \hat{\theta}_i}{\partial \hat{\theta}_i})
\]
\[ |\phi| = M_i, \quad s_{\Delta i} \theta_i^T \frac{\partial \hat{u}_i}{\partial \theta_i} < 0, \quad (9b) \]

where the projection \( P_i(*) \) is defined as follows:

\[
P_i \{ -\eta_i (1 - m_i(t)) s_{\Delta i} \frac{\partial \hat{u}_i}{\partial \theta_i} \} =
\]

\[
- \eta_i (1 - m_i(t)) s_{\Delta i} \frac{\partial \hat{u}_i}{\partial \theta_i} + \eta_i (1 - m_i(t)) s_{\Delta i} \frac{\partial |\phi|^2}{\partial \theta_i},
\]

**Theorem 1:** Consider the nonlinear interconnected system (1). Under Assumptions 1 and 2, and by adopting the fuzzy control scheme (5) and adaptive parameters laws with control gains (8)-(11), the overall control system possesses the following properties:

(i) \( |\hat{\theta}| \leq M_i, \quad x_i, u_i \in L_{\infty} \)

(ii) All \( e_i(t) \) converge to a neighborhood of zero.

**Proof:** The proof of (i) can be found in reference [3].

In the sequel, the other results will be proved.

Let choose the following Lyapunov function:

\[
V = \sum_{i=1}^N V_i = \sum_{i=1}^N \left\{ \frac{1}{2} b_i(x_i) + \frac{1}{2 \eta_i} \Phi_i^T \Phi_i + \frac{1}{2 \eta_i} \tilde{e}_i^2 \right\} \quad (12)
\]

The derivative of \( V_i \) yields

\[
\dot{V}_i = s_{\Delta i} \frac{\hat{\theta}_i}{b_i(x_i)} - \frac{\hat{b}_i(x_i) s_{\Delta i}^2}{2b_i^2(x_i)} + \frac{\theta_i \Phi_i^T \frac{\partial \hat{u}_i}{\partial \theta_i}}{2 \eta_i} + \frac{1}{\eta_i} \tilde{e}_i \tilde{e}_i .
\]

Notice that if \( |S_i| > \varphi_i \), then \( \hat{s}_{\Delta i} = \hat{s}_i \), \( u_{\Delta i} = -\text{sgn}(S_i) \). By making use of (12), one can obtain

\[
\dot{V}_i(t) \leq -\eta_i \frac{s_{\Delta i}^2}{b_i(x_i)} + s_{\Delta i} (1 - m_i(t)) [\Phi_i^T \frac{\partial \hat{u}_i}{\partial \theta_i} + \frac{1}{\eta_i} \tilde{e}_i \tilde{e}_i] + s_{\Delta i} m_i u_i^* + k_b u_{\Delta i} + s_{\Delta i} k_2 u_{\Delta i} - \frac{1}{\eta_i} \tilde{e}_i \tilde{e}_i,
\]

\[
\dot{V}_i(t) \leq -\eta_i \frac{s_{\Delta i}^2}{M_i} + s_{\Delta i} (1 - m_i(t)) [\Phi_i^T \frac{\partial \hat{u}_i}{\partial \theta_i} + \frac{1}{\eta_i} \tilde{e}_i \tilde{e}_i] - s_{\Delta i} (1 - m_i(t)) [\Phi_i^T \frac{\partial \hat{u}_i}{\partial \theta_i} + \frac{1}{\eta_i} \tilde{e}_i \tilde{e}_i] + s_{\Delta i} (1 - m_i(t)) \tilde{e}_i \tilde{e}_i + \frac{\varphi_{\Delta i}^2}{M_i} + M_i - k_b + M_i \Delta .
\]

Moreover, from the adaptive control laws (8)-(11), it can be obtained:

\[
\dot{V}_i(t) \leq -\eta_i \frac{s_{\Delta i}^2}{M_i} + I_i s_{\Delta i} (1 - m_i(t)) \frac{s_{\Delta i}}{\left| \theta_i \right|^2}.
\]

Here in \( P_i(*) \), \( I_i \) is defined as: If the first condition of (8) or (9) is true, then \( I_i = 0 \); if the second one is true, then \( I_i = 1 \). Upon consulting reference [9], it is found:

\[
I_i s_{\Delta i} (1 - m_i(t)) \frac{s_{\Delta i}}{\left| \theta_i \right|^2} \leq 0. \quad (14)
\]

Thus, the inequality (13) can be written down as
\[ \dot{V}_i(t) \leq -\eta_i \sum_{\alpha} \frac{\dot{s}_{i\alpha}^2}{M_{ji}} \leq -\eta_i \frac{\dot{s}_{i\alpha}^2}{c_i} < 0, \]
\[ \dot{V}(t) \leq -\sum_{i=1}^{N} \frac{\dot{s}_{i\alpha}^2}{c_i} < 0. \quad (15) \]

Then \( V \in L_\infty \), and \( \dot{s}_{i\alpha}, \dot{s}_{i\alpha}^2 \in L_\infty \). Moreover, by using (2), it can be shown \( e_i \in L_\infty \). Since \( X_{ji} = e_j + X_{ji} \), then \( X_{ji} \) is bounded. Due to (15) and because 
\[ V = \sum_{i=1}^{N} V_i, \] it is apparent \( V \) is monotone decreasing and inferior bounded. Hence \( \lim_{t \to \infty} V(t) = V(\infty) \) exists, and integration of (15) yields
\[ \int_{0}^{\infty} \dot{V}(t) \, dt \leq V(0) - V(\infty) < 0. \] The above inequality means that also \( s_{i\alpha} \in L_2 \). Since \( \dot{s}_{i\alpha} \in L_\infty \), by virtue of Barbalat lemma it follows \( \lim_{t \to \infty} \dot{s}_{i\alpha}(t) = 0 \), and, moreover, it can be deduced that \( |s_{i\alpha}| \leq \varphi_i \). In other words, all \( e_i(t) \) converge to a neighborhood of zero. Now, by using Eq. (5), it is easy to prove that \( u_i \) is bounded too. And, this completes the proof.

V. CONCLUSION

A new kind of the second class of fuzzy direct sliding mode control system for the class (1) of interconnected non-linear plants has been synthesized. The controller structure has the following properties: (i) There is no need to know the specific mathematic model of the control objective and of the interconnections; (ii) the fuzzy control rules can be directly utilized; (iii) the close-loop system is guaranteed to be practically globally stable in the sense that all signals are bounded within tolerance limits. Thus, a new method for dealing with unknown or uncertainty interconnections is provided. The application of fuzzy sliding mode control concept replaces the “supervisory control” [9], [10], [11].

Note that the formation of the designed control system architecture is seemingly decentralized, which may facilitate the implementation. By going deeper into the essence of the designed system, however, it can be seen that the information of all the process variables are provided to the fuzzy rules, which undoubtedly increases the dimension of the basis functions. Hence, in order to cope with “the curse of dimensionality”, the second case of the FDAC has been adopted in the designed system. The stability theorem for the presented design of this adaptive control system is an entirely new theoretical result.

It may readily be inferred from this work that the next future research is to focus on how to make good use of the properties of interconnected systems such as “similarity” and “symmetry” when present in the plant system. The latter are believed to reduce the dimension of the basis functions so as to make them depend solely on subsystems’ states, thus enhancing the decentralized control implementation. It is believed this work has a substantial significance.

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VII. REFERENCES